Rapid measurement of atomic clock-state qubits for violating Bell inequalities

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ABSTRACT

Optical clock-transitions such as the ones in Ytterbium are prime candidates for encoding qubits for quantum information processing applications due to very low decoherence rates. In this work, we investigate the challenges involved in using these prime candidates for fundamental tests of quantum mechanics. We design entangling operations for pairs of indistinguishable atoms trapped in optical tweezers, as well as determine the feasibility of rapid qubit rotation and measurement of qubits encoded in these desirable low-decoherence clock transitions. In particular, we propose multi-photon transitions for fast rotation of qubits, followed by ultrafast readout via resonant multiphoton ionization. The rapid measurement of atomic qubits is crucial for high-speed synchronization of quantum information processors, but is also of interest for tests of Bell inequalities. We investigate a Bell inequality test that avoids the detection loophole in entangled qubits, which are spacelike separated over only a few meters.

Keywords: Ytterbium, entanglement, fast single-qubit gates, Bell-inequalities

1. INTRODUCTION

Fundamental tests of quantum mechanical vs. classical predictions have played a central role in the study of quantum information.[1] Experimental tests to answer Einstein's question about the completeness of quantum mechanics have been underway for nearly thirty years,[2,3] yet none of these tests have been able to simultaneously close detection and locality loopholes to rule out a description of observed phenomena due to local hidden variables (LHV). Experiments with entangled photons remain subject to detection loopholes and enhancement arguments, as a result of low detector efficiencies.[4,5] Atomic systems can be read-out with high efficiencies as demonstrated for ions,[6] but measurements have not yet been spacelike separated, as is necessary to avoid any possibility of classical information exchange. Finally, in the case of neutral atom systems, entangling operations are difficult and have so far only been shown for large ensembles of atoms without verifying pair wise entanglement correlations.[7,8]

We propose an experimental method to test the Clauser-Horne-Shimony-Holt (CHSH) inequality[1] that simultaneously closes both the detection and spacelike separation loopholes in a neutral atom system. Moreover, we address the possibility of achieving spacelike separation in a single laboratory with atoms separated by only a few meters, as opposed to other proposals which consider atom entanglement over long distances.[9] The setting we propose is a pair of identical neutral atoms trapped using optical tweezers, allowing entanglement operation and subsequent atom-transport over a few meters. As our proposals involves identical bosonic atoms, we can use an entangling operation described elsewhere,[8,10] which is similar to the spin-exchange gate developed primarily for Fermions by Hayes *et al.*[11] By applying the proper symmetrization requirements and exploiting the identical particle nature, gates based on this exchange interaction feature an inherent robustness to errors. This allow us to design an entangling operation using optical tweezers even for atoms with not well-known interaction strengths (e.g. Yb[12]). We discuss encodings of quantum information in Yb and design fast measurement and readout schemes. The necessary short measurement times for ensuring strict Einstein-locality conditions

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could be achieved with fast rotation of qubits via multiphoton transitions and readout via photoionization. Fast measurements of about 100 ns have recently been experimentally demonstrated for Rb atoms.[13] We explore the limits to fast measurements for encoding in optical clock states of Yb which are easily spectrscopically resolved and addressed for measurements on a ∼1ns timescale.

The system we describe provides not only an excellent test bed for quantum mechanics, but also an excellent starting point for the study of small-scale quantum information processors. Clock states promise high fidelity quantum memory for small scale quantum information processing. Entangled Yb and other Group II atoms are potentially very useful for precision measurements as well as for the measurement of fundamental constants. Our work will provide practical guidance for cutting-edge experiments that consider neutral Yb for quantum information processing applications. Finally, addressing fast rotation and readout of qubits is critically important for experimental realizations of measurement-based quantum computing architectures.

2. ENCODING AND MANIPULATION OF CLOCK STATES IN YTTERBIUM

2.1 Encoding

The optical clock transitions in Group II-like atoms (such as Yb) are prime candidates for encoding quantum bits due to their extremely low decoherence rates. This allows storage of the quantum information for fairly long times and also allows transport over several meters without losing the fragile quantum information. Furthermore, the recent cooling of Yb into a Bose-Einstein condensate[14] makes quantum information processing in Yb specially tantalizing. We propose to encode qubits in the extremely long-lived ${}^1S_0$ and ${}^3P_0$ states of Yb. Yb and other Group II atoms feature a similar level structure, as shown in Fig. 1. These states can be trapped at the magic wavelength (see Porsev *et al.*[15] for Yb). At the magic wavelength, the polarizabilities and therefore the lightshift potentials for the two states are identical. This makes the encoding robust with respect to fluctuations in the trap-laser field and results in completely state-independent trapping potentials. This is particularly important for transporting atoms over long distances with minimum decoherence.

2.2 Single-qubit gates

Low decoherence rates in even isotopes of Yb are due to the fact that electric dipole one- and two-photon transitions between ${}^1S_0$ and ${}^3P_0$ states are dipole- and parity-forbidden, respectively. Whereas forbidding these transitions yields desirably low decoherence rates, it also presents a major challenge to fast coherent manipulation and measurement of qubits for quantum information processing. To overcome this challenge, we propose

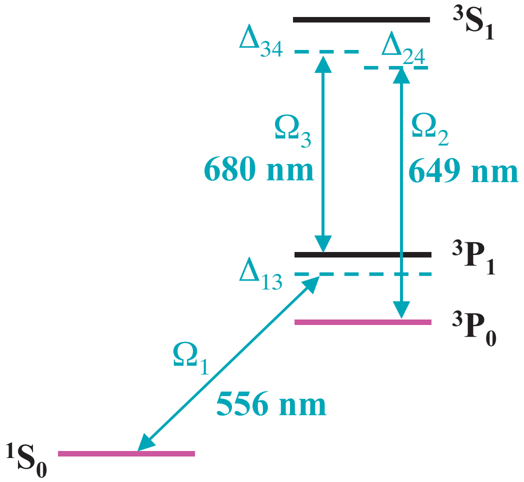

Figure 1. Energy levels of Yb and three-photon transition for manipulation of the qubit encoded in ${}^1S_0$ and ${}^3P_0$.

to use a coherent, three-photon transition (see Fig. 1) to perform single qubit operations.[16] Since the transition we consider is both one- and two-photon forbidden, a three-photon transition is necessary. This can be accomplished utilizing the excited $^3S_1$ and $^3P_1$ states as proposed by Hong *et al.*[16] The three transitions $^1S_0 \rightarrow {}^3P_1$, $^3P_1 \rightarrow {}^3S_1$, and $^3S_1 \rightarrow {}^3P_0$ are electric-dipole allowed and the corresponding transition matrix element can be found in Porsev *et al.*[17] Since three laser beams can always be arranged in a plane such that the transferred recoil cancels, this three-photon transition has the added benefit of being recoil-free,[16] thus avoiding further decoherence effects. Otherwise, for transitions between states that differ by an optical energy, there is always substantial recoil imparted on the atom, which could lead to a partial measurement of the internal state destroying any entanglement. The fidelity for this three-photon single-qubit rotation is limited due to the short-lived intermediate $^3S_1$ state which primarily decays to the $^3P_1$ state. For large detunings (i.e., much larger than the Rabi frequency and decay rates) the effective Rabi frequency and detuning can be estimated via

$$\Omega_{\text{eff}} = \frac{\Omega_1 \Omega_2 \Omega_3}{4\Delta_{13}\left(\Delta_{13} + \Delta_{34}\right)} \tag{1}$$

and

$$\Delta_{\text{eff}} = \left(\Delta_{13} + \Delta_{34} - \Delta_{24}\right) + \frac{\Omega_1^2}{4\left(\Delta_{24} - \Delta_{34}\right)} - \frac{\Omega_2^2}{4\left(\Delta_{13} + \Delta_{34}\right)} . \tag{2}$$

Here, Ω_k are the bare Rabi-frequencies for each of the three transitions and Δ_{ij} are the detunings between states $|i\rangle$ and $|j\rangle$. We model this transition using the master equation using the Liouvillian matrix given in Hong *et al.*[16] Numerical simulation of Rabi oscillations between the two clock states are shown in Fig. 2. Note that detunings have been carefully chosen to be red detuned to minimize transitions to ionized states. Nevertheless, the high power of laser pulses applied could result in ionization of atoms during the single qubit gates since, for Yb, $\lambda_1 = 556$ nm is still shorter than the ionization wavelength $\lambda_{\text{ion}} = 563$ nm. A more careful calculation including these additional errors for Yb is left for future investigation.

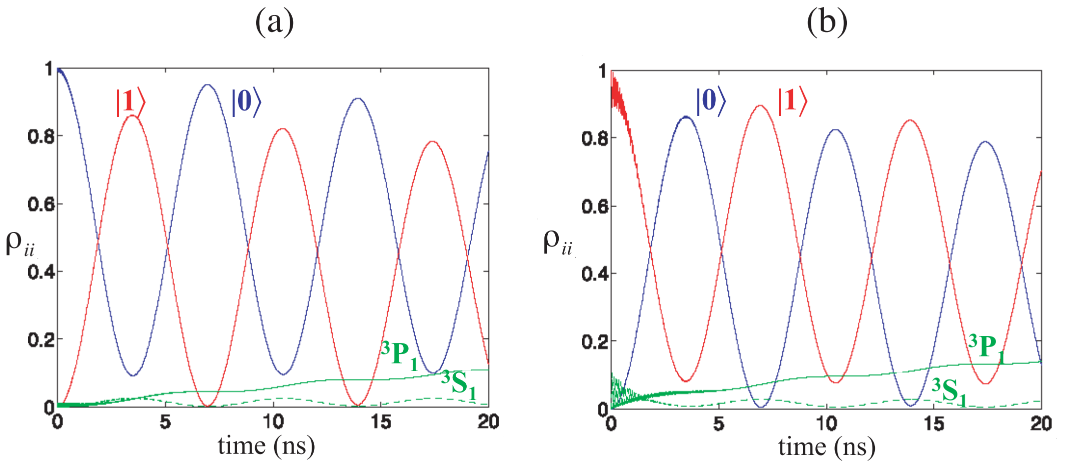

Figure 2. Rabi oscillations between clock states for initial state in the $^1S_0$ state (a) and $^3P_0$ state (b).

2.3 Detection via photoionization

The fast coherent rotation of qubits is followed by the fast readout of the $^3P_0$ state via multi-photon ionization, which is possible on nanosecond or even picosecond time scales. We again utilize the $^3S_1$ excited state. Photoionization can be accomplished in a two-step process: $^3P_0$ to $^3S_1$ followed by a final ionization step. The ionization threshold from the $^3S_1$ state is563 nm for Yb and the ionization probability can be further enhanced by carefully selecting an ionization wavelength in resonance with autoionizing states in Yb.[18] The main errors in this read-out scheme are due to population in the $^3P_1$ to $^3S_1$ states. Any population in $^3P_0$ and $^3S_1$ will count as logic $|1\rangle$ (ionized) during read-out; population in $^1S_0$ and $^3P_1$ will count as logical $|0\rangle$ (not ionized). Due to extremely high detection efficiency of proportion detectors in detecting single ions,[19] detection of atoms with close to 100% efficiency is possible in this multi-photon ionization scheme proposed here. This forms the foundations for the simultaneous closing of both the detection and spacelike separation loopholes.

2.4 Bell state creation

As described in Sec. 2.1, atoms can be trapped at the magic wavelengths in completely state insensitive potentials. Here, we imagine two atoms trapped in two optical tweezers that are brought together and separated adiabatically. When the tweezers are overlapping, atoms undergo cold collisions and a state-dependent phaseshift is acquired. Due to the identical particle nature, we can design a universal controlled phase gate based on spin-exchange gate developed primarily for Fermions by Hayes *et al.*[11] This exchange interaction has recently been experimentally demonstrated for identical bosonic Rb atoms.[8] By applying the proper symmetrization requirements to internal and motional, one can show that a significant internal-state dependent phase shift can always be acquired.[10] Even in the case of Yb, where the essential interaction strengths and scattering lengths are not well known, this enables the design of a entangling operation that is robust against noise and works for a large range of parameters. This universal entangling gate provides the necessary ingredient for the creation of an entangled EPR pair and thus enables fundamental tests of quantum mechanics.

3. TESTS OF BELL INEQUALITIES

For a fundamental test of quantum mechanics vs. local hidden variable models, we consider a test of the CHSH inequality.[1] Local hidden variable models predict an expectation value of the Bell-operator, $\langle B\rangle$, of less than 2:

$$\langle B\rangle = \langle QS\rangle + \langle RS\rangle + \langle RT\rangle - \langle QT\rangle \leq 2\,. \tag{3}$$

For a maximum violation of the Bell-inequality, the measurement operators $Q = Z$, $R = X$, $S = X + Z$, and $T = X - Z$, where X and Z are the Pauli operators. The measurement operators are obtained through a change of measurement basis by the unitary rotation operators $Q = U_Q^{\dagger} Z U_Q$. The corresponding rotations are $U_Q = 1$, $U_R = R(\pi/2)$, $U_S = R(3\pi/4)$, $U_T = R(\pi/4)$ with the standard rotation operator $R(\theta)$.

To test the CHSH inequality, simultaneously avoiding the detection and spacelike separation loopholes, we prepare an entangled EPR state via a controlled phase gate using the entangling operation described above. The entangled atoms are then transported over several meters. Spacelike separation of qubit measurements at this distance demands synchronous measurements on a nanosecond timescale. In this time window, the measurement basis is chosen randomly, qubit are rotated to reflect the chosen measurement bases, and finally the population of one of the qubit states is readout via a multiphoton ionization process. The rotation of measurement bases is achieved via a coherent coupling of the qubit states via three-photon transitions as described in Sec. 2.2. The presence of the ion, i.e., the freed electron, will be detected via a standard proportional detector.

Note that unlike in photon experiments, all events are counted here and no singles are discarded. This avoids possible enhancement arguments and the detection loophole. Any losses and errors (e.g., scattering into states outside the logical basis and errors during the rotation of bases) are counted as events and will lower the expectation value of the Bell operator. A detailed calculation of the Bell inequality violation for imperfect rotation, including errors in rotation and readout as discussed above, is shown in Fig. 3. This calculation shows that in order to achieve an average value of the Bell-operator of larger than 2, as required for a violation, measurements on a time scale of more than 1 nanosecond are necessary using reasonable limits to available laser power. A 1 ns measurement implies that a separation between qubits on the order of one meter is sufficient to enforce the spacelike separation condition and hence close the locality loophole. Such separations between atomic qubits should be feasible using the optical tweezers set-up described above. Our calculations thus show that a Bell-inequality test that closes both important loopholes simultaneously may be possible to realize using Group II atoms in a single laboratory.

4. SUMMARY

In this work, we have proposed experiments that should enable the simultaneous closure of both the detection and locality loopholes for Bell-inequality tests. Here, we consider encoding in optically separated atomic clock states in Yb that feature extremely low decoherence rates, fast qubit manipulation, and rapid readout on a nanosecond

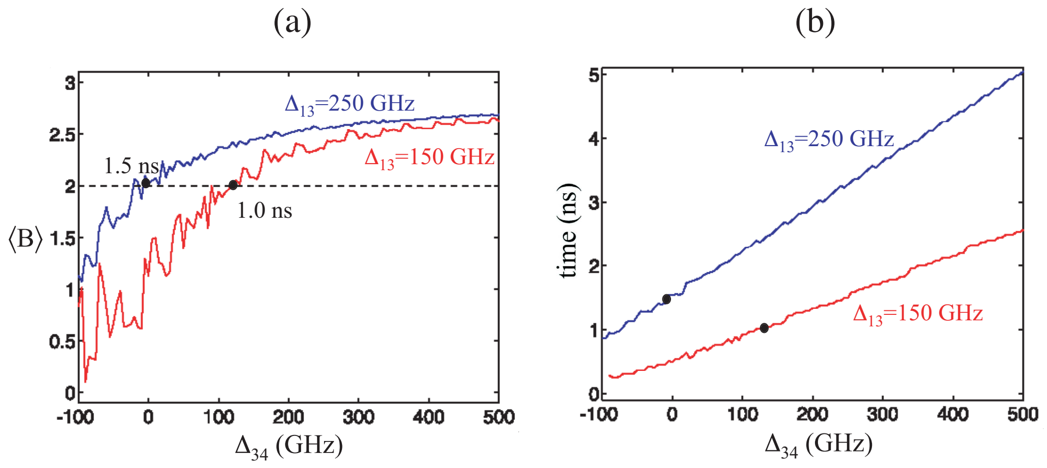

Figure 3. (a) Expectation value of the Bell operator for imperfect single-qubit rotations as a function of detunings. Bell inequality violation occurs above $\langle B\rangle = 2$ with qubit manipulation times longer than 1 nanosecond ($\Delta_{13} = 150$ GHz) and 1.5 nanoseconds ($\Delta_{13} = 250$ GHz). The peak laser pulse intensity has been limited to a reasonable value of 10^9 W/cm^2 ($\approx$1kW pulse peak power focused onto $(10\mu\text{m})^2$ spot). Shorter measurement times can be achieved by reducing the detuning resulting in errors, which reduce the possible Bell-inequality violation. (b) Time for measurement as a function of different detunings.

timescale. Furthermore, this study sets the groundwork for future exploration of measurement-based computation and synchronization of small scale quantum computers. Future studies include a more detailed analysis of Yb and in depth consideration of other group II atoms such as Sr, which might present further advantages.

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