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An experimental realization of the quantum δ-kicked rotor

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Abstract. We study experimentally quantum effects in atomic motion for a classically chaotic regime. A standing wave is pulsed on periodically in time, and the resulting atomic momentum is measured. Momentum grows diffusively until the quantum break time, after which dynamical localization is observed. Quantum resonances are observed for certain pulse durations.

1. Introduction

The study of quantum systems that are classically chaotic has been a topic of active research in recent years. Examples include atoms in strong fields [1, 2], electrons in a one-dimensional metal [3], scattering of electrons in mesoscopic structures [4], and molecular excitation [5]. The paradigm theoretical system in this field has been the quantum δ-kicked rotor (QKR), and a broad range of universal effects have been predicted in this model. In this paper we report the first direct experimental realization of the QKR [6], and the observation of dynamical localization, which is a key prediction of the theory. Our system consists of laser-cooled sodium atoms that are exposed to a one-dimensional periodic potential formed by a standing wave of near-resonant light. The standing wave is pulsed on periodically in time with the duration of the pulse much shorter than the period between pulses, and atomic momentum is measured as a function of interaction time and the pulse period. We observe diffusive growth of energy until a 'quantum break time' after which dynamical localization sets in [6–8]. We also observe 'quantum resonances' when the pulse period is a multiple of the natural period. The conceptual simplicity of this system and the high degree of control over the interaction potential make it an ideal testing ground for the field of quantum chaos.

2. Model system

We begin with a two-level atom interacting with a near-resonant laser beam. If the atom is excited into the upper state, it will then spontaneously emit a photon and the atomic recoil will be in a random direction. When the laser is sufficiently detuned from resonance, a coherent scattering process dominates over the absorption and spontaneous emission. This coherent scattering occurs along the direction of the beam. In a standing wave of light formed from two counterpropagating laser beams, scattering from one beam into the other leads to a two-photon recoil, which corresponds to a velocity change of 6 cm s^{-1} for sodium atoms. In this limit, the atom remains in the (internal) ground state, and changes its momentum in units of two recoils. This system is described by a one-dimensional Hamiltonian

$$H = p^2/2M - V(t)\cos[2k_L x - \phi(t)] \tag{1}$$

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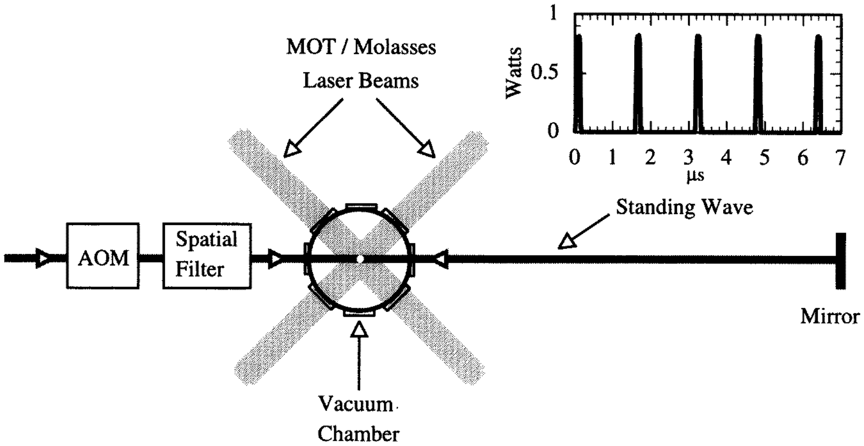

Figure 1. Illustration of the experimental set-up. Sodium atoms are trapped and cooled in a MOT. The trapping beams are then turned off, and a second, detuned laser is turned on in a train of short pulses. The intensity is controlled with the AOM, the beam is spatially filtered, overlapped with the atoms and retroreflected from a mirror to form a standing wave. The pulse sequence is shown as an inset in the figure. The pulse duration is 110 ns and the period between pulses is 1.58 μs for this trace. These are the pulse parameters used in all the figures shown.

where $V(t)$ is the AC Stark shift [9] that is proportional to the laser intensity and inversely proportional to the detuning of the laser from atomic resonance, and k_L is the wavenumber. The period of this potential is half the laser wavelength. We consider the general case where the amplitude, $V(t)$, and phase, $\phi(t)$, are time dependent.

Our first measurements of momentum transfer in a time-dependent potential were done with a periodically modulated standing wave. In this case the Hamiltonian had the following form [8]:

$$H = p^2/2M - V\cos[2k_L(x - \Delta L \sin\omega_m t)]\,. \tag{2}$$

The standing wave's position is modulated at a frequency ω_m with an amplitude ΔL. This Hamiltonian describes a periodically driven pendulum. Although this system can be locally described by the QKR, for the regime that is most easily accessible experimentally, it has the complication of island structure in the classical phase space associated with zeros of Bessel functions [10, 11]. Dynamical localization was predicted for this system in a regime that is predominantly chaotic (classically), and was observed in our experiments [10].

To obtain a more direct experimental realization of the QKR we pulse the standing wave on periodically in time, instead of modulating the phase. This system is described by the Hamiltonian

$$H = p^2/2M - Vf(t)\cos(2k_L x) \tag{3}$$

where $f(t)$ is a sequence of N pulses with period T and duration τ, as shown in figure 1. A similar model was previously proposed for rotational excitation of diatomic molecules [5]. For small momenta, motion across the standing wave potential (while the pulse is on) is negligible. This results in an integrated impulse which is the same as that due to a delta function, and can be directly compared with the QKR stochasticity parameter κ. A classical phase portrait for this Hamiltonian (figure 2) exhibits global chaos for momenta in a bounded regime. The boundary in this system is due to the fact that for large enough atomic momenta the atom can move a significant portion of a well period, leading a smaller

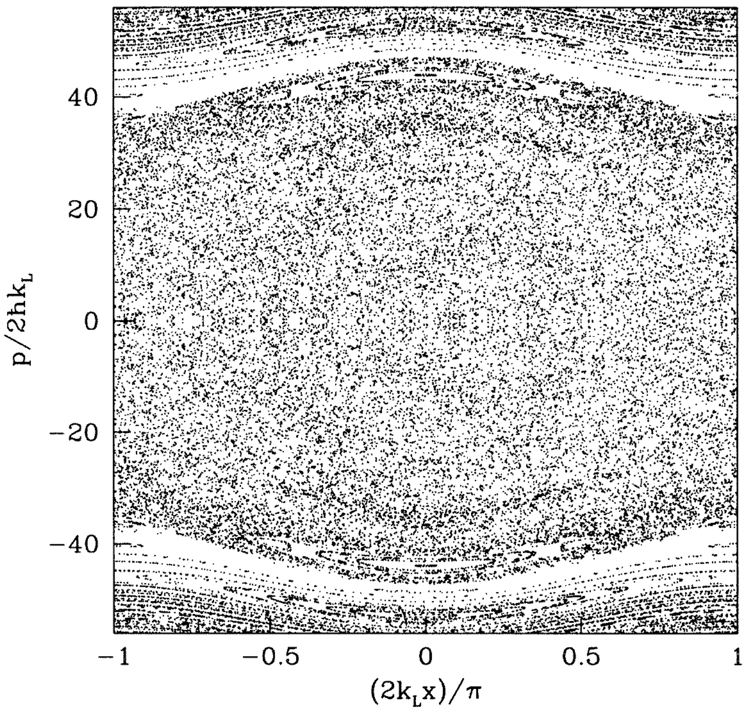

Figure 2. Classical phase portrait for the parameters of the experiment. The vertical axis is momentum in units of two recoils. The horizontal axis is position in units of well period. For momenta in the region $[-6, 6]$ the impulse approximation is good and the system is nearly identical to the classical kicked rotor. The effective stochasticity parameter is $\kappa = 11.6$.

effective kick. The associated variation in the stochasticity parameter results in islands of stability and KAM surfaces as the boundary is approached. By making the duration of the pulse shorter, the delta-function approximation is valid for larger values of momenta. In the work described here, the momentum remained localized in a regime which is far from the boundary so that the variation in the impulse was small. In the quantized model, x and p are operators satisfying the commutation relation $[x, p] = -\mathrm{i}\hbar$. In scaled dimensionless units, $\phi = 2k_L x$, $\rho = (2k_L T/M)p$, and the scaled Planck's constant $\bar{k} = 8\omega_r T$ where ω_r is the recoil frequency ($\omega_r/2\pi = 25$ kHz for sodium).

3. Experimental results and discussion

To study this time-dependent interaction experimentally, there are three important components: initial conditions, interaction potential and detection of momentum transfer. A schematic of the experimental set-up is shown in figure 1. Our initial conditions are a sample of ultra-cold sodium atoms which are trapped and laser-cooled in a magneto-optic trap (MOT) [9]. The atoms are contained in an ultra-high vacuum glass envelope at room temperature. The trap is formed using three pairs of counterpropagating, circularly polarized laser beams (2.0 cm beam diameter) which intersect in the middle of the glass envelope, together with a magnetic field gradient which is provided by current-carrying wires arranged in an anti-Helmholz configuration. These beams originate from a dye laser that is locked 20 MHz to the low-frequency (red) side of the $(3\mathrm{S}_{1/2}, F = 2) \longrightarrow (3\mathrm{P}_{3/2}, F = 3)$ sodium

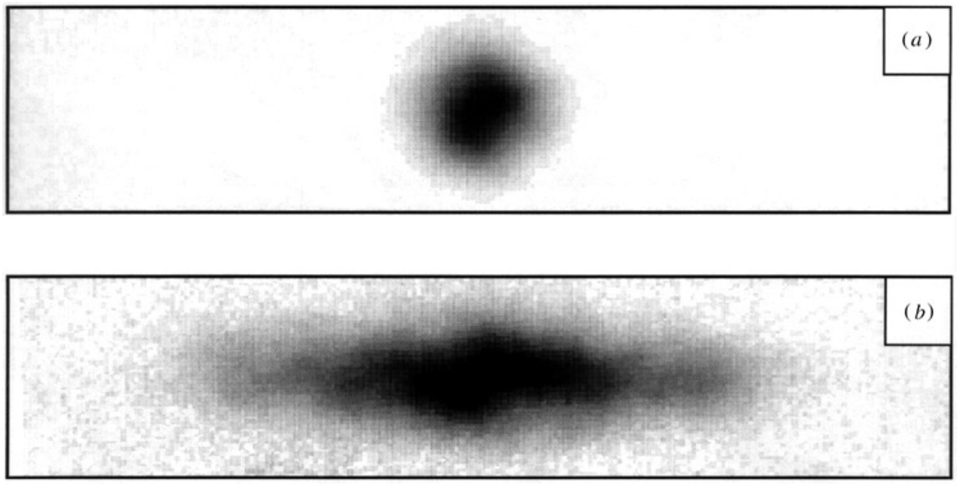

Figure 3. Two-dimensional atomic distributions after free expansion. (*a*) Initial thermal distribution with no interaction; (*b*) localized distribution after interaction with the potential.

transition at 589 nm. Approximately 10^5 atoms are trapped in a cloud which has an RMS size of 0.12 mm, with an RMS momentum spread of $4.6\hbar k_L$. The interaction potential is provided by a second dye laser that is tuned to 5 GHz red of the resonance. The output of this laser is aligned through a fast acousto-optic modulator (25 ns rise time) which is driven by a pulse generator. This device controls the laser intensity in time. The beam is then spatially filtered to ensure a Gaussian intensity profile, and is centred on the atoms. The beam is retro-reflected from a mirror outside the vacuum chamber to create a standing wave.

The detection of momentum is accomplished by allowing the atoms to drift in the dark for a controlled duration, after the interaction with the standing wave. Their motion is 'frozen' by turning on the optical trapping beams in zero magnetic field to form optical molasses [9]. The position of the atoms is then recorded via their fluorescence signal on a charged coupled device (CCD) and the time of flight is used to convert position into momentum. The entire sequence of the experiment is computer controlled.

In figure 3, typical 2D images of atomic fluorescence are shown. In figure 3(*a*) the initial MOT was released, and the motion was frozen after a 2 ms free-drift time. This enables a measurement of the initial momentum distribution. The distribution of momentum in figure 3(*a*) is Gaussian in both the horizontal and vertical directions. The vertical direction is integrated to give a one-dimensional distribution as shown in figure 4(*a*). In figure 3(*b*), the atoms were exposed to a sequence of kicks along the horizontal axis which imparted momentum in that direction. In that case the vertical distribution remains Gaussian, but the horizontal distribution becomes exponentially localized due to the interaction potential, as shown in figure 4(*b*). The parameters for this case are $\bar{k} = 2.0$ and $\kappa = 11.6$.

By varying the number of kicks and measuring the resulting momentum distribution, we have mapped out the evolution in time. We observe an initial growth in energy which is linear in time corresponding to chaotic diffusion until a quantum break time after which we observe dynamical localization. The experimental results are in good agreement with no adjustable parameters with a direct integration of Schrödinger's equation, and with the predictions for the quantum standard map [6, 12]. The measured momentum distribution

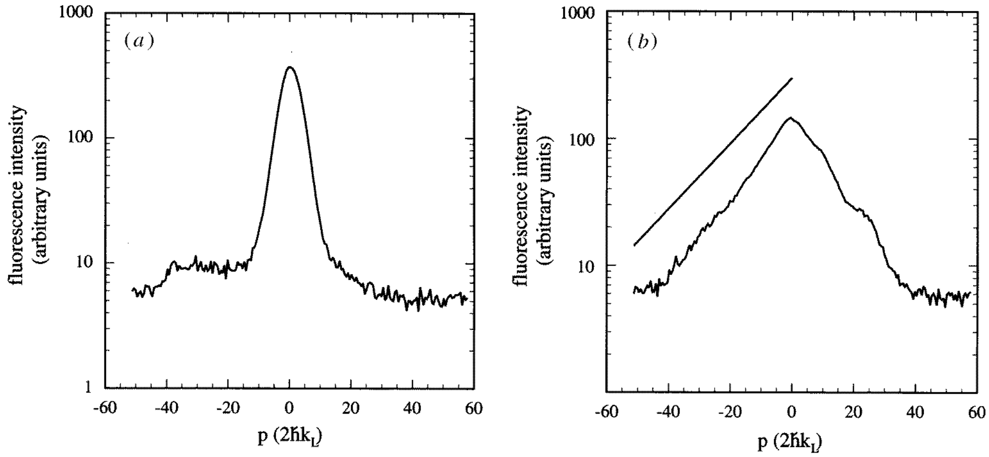

Figure 4. One-dimensional atomic momentum distributions. They were obtained by integrating along the vertical axes of the 2D distributions in the previous figure. The horizontal axes are in units of two recoils, and the vertical axes show fluorescence intensity on a logarithmic scale (*a*) Initial thermal distribution with no interaction; (*b*) localized distribution after interaction with the potential. The exponential lineshape is clearly seen and is in close agreement with the theoretically predicted localization length (straight line).

shown in figure 4(*b*) is exponentially localized and the straight line shows the prediction for dynamical localization in the kicked rotor [12]. This result is well within the quoted 10% error bar in κ arising from uncertainty in absolute laser intensity calibration.

In the QKR there is a well defined period of kicks. The evolution can therefore be separated into kicks with free propagation in between. When the kicking period is adjusted so that the free evolution phase, $p^2T/2\hbar M$, is a multiple of 2π, a unique situation occurs that does not have a classical analogue. This condition is equivalent to $T = 2\pi n/4\omega_r$ for integer n. We observe a dramatic change in the momentum distribution for integer and half-integer values of n ranging from 0.5 to 5 in 0.5 steps. The half-integer resonances correspond to a quantum phase of π, resulting in an alternating sign of adjacent kicks. The momentum profile on resonance is substantially *narrower* than the exponentially localized distribution. This is considerably different from the textbook prediction and has been shown to result from non-plane-wave initial conditions [13]. The observed quantum resonances are in good agreement with the modified theoretical analysis [13].

4. Conclusion

We have shown that atom optics can be a simple testing ground for the field of quantum chaos. In the work described here, a direct experimental realization of the quantum δ-kicked rotor was reported. This work should open many new possible directions for future studies of the role of quantum mechanics in driven systems.

Acknowledgments

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