NEW LIGHT ON QUANTUM TRANSPORT

Atoms moving in an accelerating optical lattice exhibit quantum behavior such as Bloch oscillations, Wannier–Stark ladders and tunneling—phenomena usually associated with electrons in a crystalline solid.

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Consider the following experiment: Take a carton of eggs, open the lid and accelerate the carton with a sudden jerk. If you try this at home, you will find that the outcome strongly depends on the magnitude of the acceleration (the authors are not responsible for the results).

With this classical picture in mind, we can ask, What is the corresponding behavior of a microscopic particle in an accelerating periodic potential? It is helpful to transform this potential to the comoving frame of reference, since that is what the particle "sees." The result is very simple: a tilted, or "washboard," potential, as shown in figure 1a. The tilt is proportional to the acceleration. Classical mechanics predicts that a particle is either trapped or not, depending on the initial conditions. A trapped particle will end up confined within a single well of the potential; one that's not trapped will roll down the slope. (In the lab frame, this corresponds to the particles either being carried along with the potential or getting left behind.) Beyond a certain acceleration, all particles will just roll down the hill, as illustrated in figure 1b. When quantum mechanics is used to predict the particle motion, however, the results can be strikingly different and counterintuitive. In the regime of quantum transport, motion is dominated by quantum interference and tunneling.

The system of quantum particles moving in a periodic potential has long been a basic model for electrons in crystalline solids. The acceleration is replaced in this case by a DC electric field, but the effect is the same. As early as the 1930s, Felix Bloch and Clarence Zener combined the ideas of the newly founded quantum mechanics and translational symmetry of lattices to show that stationary states of electrons in a lattice are plane waves modulated by periodic functions of position.[1] A wavepacket that is initially localized in space will spread by way of resonant "Bloch tunneling," and will eventually become delocalized. The quantized energy levels are broadened into energy bands due to this tunneling process, as shown in figure 1c.

When a tilt is imposed on the periodic potential, the translational symmetry is broken, and the initial wavepacket can remain localized because Bloch tunneling is suppressed. The degree of localization increases with the magnitude of the tilt. The particle motion, however, is periodic in time due to repeated Bragg scattering, a phenomenon known as Bloch oscillations. The Bloch period is $\tau_B = h/Fd$, where h is Planck's constant, d is the lattice spacing and F is the force on the particle ($F = eE$ for an electron in an electric field E; $F = ma$ for a particle of mass m in a potential undergoing acceleration a). In the case of the accelerating potential, the phenomenon of Bloch oscillations means that the particle will track the accelerating potential only on the average, and will display periodic oscillations in space and in momentum. These oscillations can extend over many periods of the lattice and do not have a classical analog. For the case of electrons in an ideal lattice, an applied voltage should yield an AC current corresponding to the Bloch oscillations, but no DC current. This quantum effect is in sharp contrast to our everyday (room temperature) experience where we measure DC currents that obey Ohm's law.

In the early 1960s, Gregory Wannier proposed that Bloch electrons in a constant electric field have an energy spectrum consisting of sets of equally spaced energy levels,[2] now referred to as Wannier–Stark ladders, with level spacing given by h/τ_B. This effect constitutes a natural extension of the Stark effect in atoms, in which a degenerate electronic level splits into equally spaced levels under an electric field. The Wannier–Stark ladders marked such a dramatic departure from the Bloch bands that this prediction was very controversial.[3]

As the tilt of the potential becomes comparable to one well depth per lattice spacing, a new tunneling process becomes important. This effect, known as Landau–Zener tunneling, corresponds to interband transitions. It interrupts the coherent Bloch oscillations and broadens the linewidths in the Wannier–Stark ladder. In the accelerating lattice, the manifestation of this effect should be the escape of particles from the potential, as illustrated in figure 1d. This tunneling loss should be the ultimate limit for the particle accelerator, and should occur for tilts that are much smaller than the classical limit (figure 1b). For electrons in a solid, the wavefunction should become delocalized as the electric field is increased, an effect known as Zener breakdown.

Bloch oscillations and Wannier–Stark ladders have not been observed in a crystalline solid because scattering

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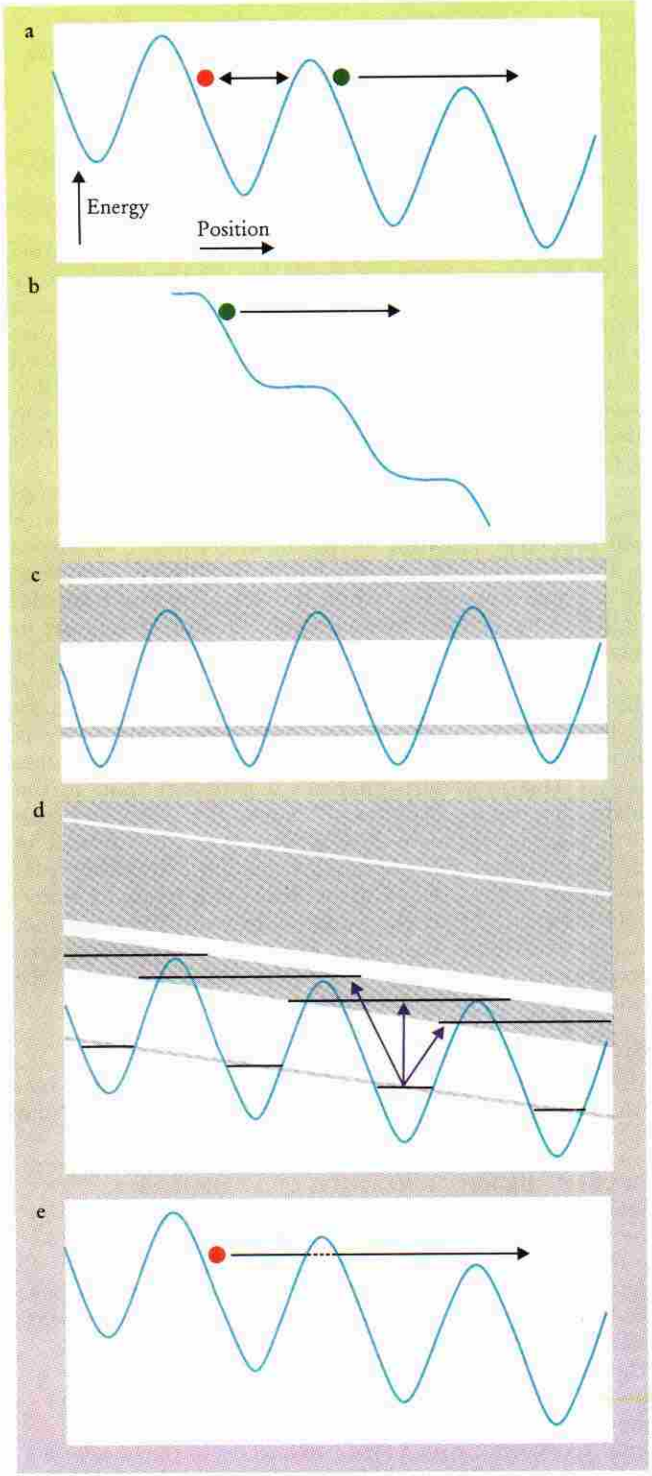


FIGURE 1. ACCELERATED PERIODIC POTENTIALS. **a:** In the comoving frame of reference, the potential is tilted. A classical particle either becomes stably trapped within a single well (red) or escapes down the hill (green). **b:** With enough tilt, there are no local minima and no particles are trapped. **c:** Quantum mechanics predicts the formation of Bloch bands in a periodic potential. The states corresponding to these bands involve particles spread across multiple wells of the potential. **d:** In a tilted potential, the Bloch bands are broken up into Wannier–Stark ladders of states. Arrows indicate resonant excitations used by the Texas group to observe the ladders. **e:** Quantum mechanics also allows Landau–Zener tunneling of a particle from a tilted potential.

by impurities, phonons and other particles effectively prevents the completion of even a single period of Bloch oscillation. Another problem is that the natural lattice spacing is very small (less than a nanometer), requiring enormous electric fields to obtain the substantial tilt of the potential needed for small Bloch periods. The situation becomes much more favorable in clean superlattices that are fabricated by epitaxial growth of GaAs and GaAlAs. In a superlattice, alternating regions of GaAs and GaAlAs produce a periodic potential seen by the electrons. The lattice constant of these structures can be tens of nanometers, yielding a much shorter Bloch period under the same electric field. In the late 1980s, Wannier–Stark ladders were seen in optical absorption and photocurrent measurements of superlattices. Evidence for Bloch oscillations was seen in the time domain using the technique of four-wave mixing with picosecond lasers, and the observation of Zener breakdown was reported. The prospect of utilizing such a "Bloch oscillator" as a source of terahertz electromagnetic radiation has stimulated much work in recent years. (For a recent review, see the article by Emilio Mendez and Gérald Bastard, PHYSICS TODAY, June 1993, page 34.)

These results represent an important breakthrough in the study of quantum transport of electrons, but many challenges remain. Dissipation and elastic scattering by impurities are still a central problem limiting the coherent evolution required for quantum transport. Such effects are evident in the broad lineshapes that smear out spectral and temporal detail. The control of initial conditions is difficult in condensed matter experiments, and direct measurement of electron motion is not possible. These difficulties provide motivation to identify a new testing ground for these striking quantum phenomena that can complement the superlattice experiments.

Quantum transport in optical lattices

With the recent development of techniques for laser manipulation and laser cooling of atoms,[4,5] new systems have emerged to study Bloch oscillations, Wannier–Stark ladders and Landau–Zener tunneling. These systems use atoms instead of electrons and a periodic light field instead of the periodic crystalline potential. The advantages of this approach are precise initial state preparation and final detection, negligible dissipation or defects and the possibility for time-resolved measurements of quantum transport. Two groups, one at the Ecole Normale Supérieure (ENS) in Paris and the other at the University of Texas at Austin, have recently observed Bloch oscillations of atoms and the Wannier–Stark ladder.[6,7] The experimental work in Paris was done by Maxime Ben Dahan, Ekkehard Peik, Jakob Reichel, Isabelle Bouchoule and Christophe Salomon, in collaboration with theorist Yvan Castin. The experimental work in Austin was done by Cyrus Bharucha, Kirk Madison, Steven Wilkinson, Patrick Morrow and Mark Raizen, in collaboration with the condensed matter theory group of Qian Niu (together with student Georgios Georgakis and visitor Xian-Geng Zhao), and with theorist Bala Sundaram.

The experiments have several common features:

▷ A gas of laser-cooled atoms (sodium in Austin and cesium in Paris), provides very well defined initial conditions. The atomic samples are sufficiently dilute that atom–atom interactions are negligible. The experiments therefore probe single-atom phenomena, although they are performed on an ensemble of atoms.

▷ The light field is created by a laser standing wave made of two counterpropagating, equal-intensity waves.

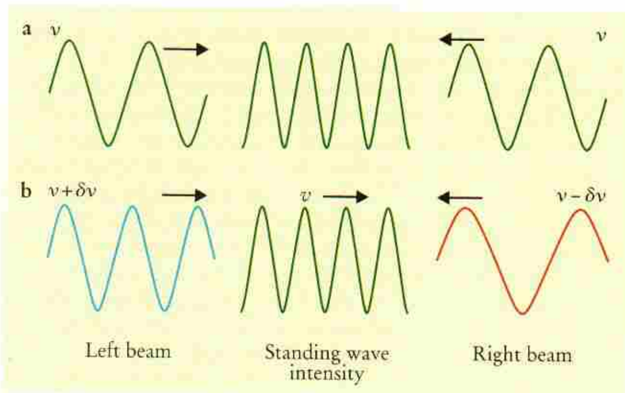
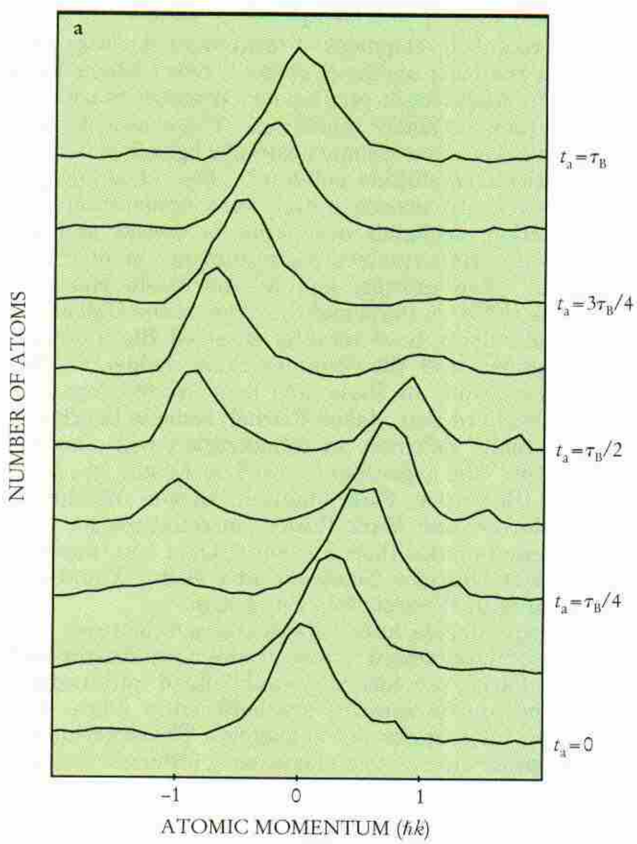


FIGURE 2. LASER BEAM CONFIGURATIONS to produce a standing wave that is (**a**) stationary or (**b**) moving to the right with velocity $v = \lambda\delta\nu$ in the laboratory frame. An accelerating standing wave is produced by increasing the frequency difference $2\delta\nu$ linearly in time. In the Texas experiments, for example, a linear ramp of 4 MHz in 800 μs created a potential accelerating at 1500 m/s^2.

When these have exactly the same frequency, the potential is stationary.

▷ The standing wave is accelerated by chirping the frequency difference of the two counterpropagating waves. This method is commonly used with resonant light in atomic fountain clocks to launch atoms upward.[5]

First consider the stationary case. The light intensity along the standing wave is of the form $I_0 \sin^2(kz)$, where k is the laser wavenumber ($k = 2\pi/\lambda$, where λ is the optical wavelength), and has a periodicity $d = \lambda/2$. For a sufficiently large detuning between the laser and the atomic transition frequency, the atoms remain in the ground state and simply experience a potential $U(z)$ that is proportional to the intensity, and is therefore of the form $U_0 \sin^2(kz)$. The well depth U_0 in the experiments was typically several times the one-photon recoil energy, $E_R = \hbar^2 k^2/2m$. (E_R is the energy an atom would have if it acquired the momentum $\hbar k$ of one photon of the light field.) In the two transverse directions, the atoms undergo nearly free-particle motion because the variation in laser intensity is negligible over the size of the atomic cloud.



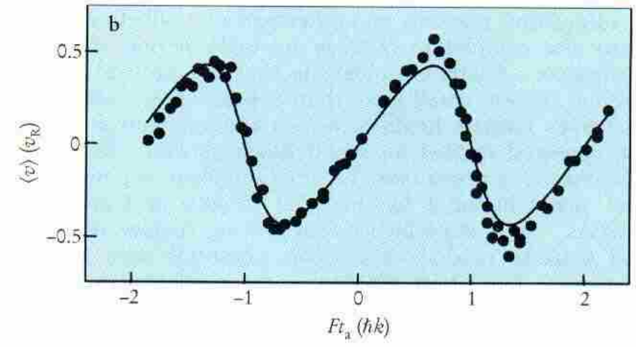
The periodic potential leads to energy bands separated by bandgaps, as shown in figure 1c. The band structure can also be represented in the reciprocal lattice as a dispersion relation between energy and quasi-momentum q (also known as the crystal momentum). Since the optical potential is created by light, an alternative description based on a time-dependent redistribution of photons can also be given,[8,9] and it has been shown that this approach is fully equivalent to the Bloch theory.

To understand how the potential is accelerated, suppose first that, instead of forming the standing wave with two counterpropagating waves having equal frequencies ν_0, the wave coming from the left is upshifted in frequency by a small amount $\delta\nu$, while the wave coming from the right is downshifted by the same amount, as shown in figure 2. In the reference frame moving to the right at a velocity $v = \lambda\delta\nu$, the two waves are Doppler shifted to the same frequency and the periodic potential is stationary in this frame. Suppose now that over a time t_a, $\delta\nu$ is increased linearly with time from 0 to a maximum value $\delta\nu_{max}$. This procedure produces a potential that, in the laboratory frame, is uniformly accelerated with an acceleration proportional to $d(\delta\nu)/dt$ during t_a. In contrast to resonant atom–light interactions, the large detuning from resonance in these experiments leads to a coherent atom–field interaction that is dissipation-free. In the comoving, accelerated frame, the atoms experience an inertial force proportional to the acceleration, in addition to the force resulting from the periodic potential.

Bloch oscillations

Although they use similar physical systems, the two groups employ different methods of state preparation and

FIGURE 3. BLOCH OSCILLATIONS OF ATOMS in the fundamental band. **a:** Momentum distribution of atoms in the accelerated frame for acceleration times t_a ranging from 0 to the Bloch period, $\tau_B = 8.2$ ms. Bragg reflection of the matter wave occurs when the atomic momentum p approaches the Bragg condition, $p = \hbar k$. (The small peak in the right wing of the first five spectra is an artifact created by a stray reflection of the Raman beams on the cell windows). **b:** Measured mean atomic velocities (circles) in units of the photon recoil velocity v_R compared to theory (solid line). Both sets of data are for a light potential depth $U_0 = 2.3\,E_R$ and acceleration $a = -0.85$ m/s^2. (From ref. 6.)



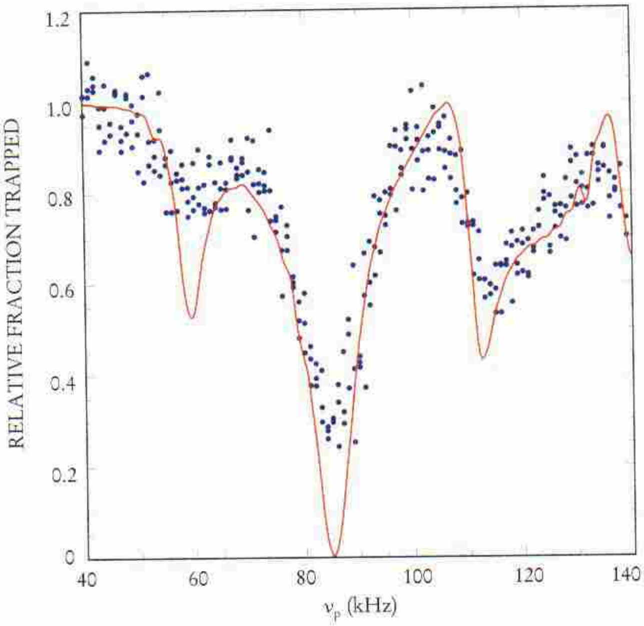


FIGURE 4. WANNIER–STARK LADDER RESONANCES obtained by adding a phase modulation of frequency ν_p to the accelerating standing wave, and measuring the number of surviving atoms. For this spectrum, the experimental parameters are $U_0 = 3E_R$ and $a = 1570$ m/s^2. The modulation amplitude is 1.5% of the well period. The solid line is a quantum numerical simulation that uses the experimental parameters. (From ref. 7.)

measurement, and explore somewhat different parameter regimes. The approach of the Paris group is best suited to measurements in the time domain (allowing observations of Bloch oscillations), while the focus of the Austin group is on measurements in the frequency domain (allowing studies of the Wannier–Stark ladders).

The Paris experiments are all performed with solid state diode lasers. The first step is to trap cesium atoms in a magneto-optical trap.[4,5] The atoms are then further cooled in one dimension to 12 nanokelvins using stimulated Raman cooling.[10,11] The corresponding root-mean-square momentum spread δp of the atoms along the direction of the periodic optical potential is then one-quarter of the momentum $\hbar k$ of a single photon from the standing wave. The Heisenberg uncertainty relation for position and momentum then implies that the coherence length of the particles $h/\delta p = 8\pi/k$ extends over several periods d of the optical potential ($d = \pi/k$). This situation is very favorable for the study of quantum effects such as Bloch oscillations and tunneling between adjacent sites of the potential.

After the cooling phase, the stationary light potential is switched on adiabatically, preparing a statistical mixture of Bloch states in the ground energy band centered around $q = 0$ and having a quasi-momentum width $\delta q = k/4$. The standing wave is then accelerated for an adjustable time t_a, simulating the external force in the comoving frame. Finally, the optical potential is abruptly switched off and the atomic momentum distribution is measured with a resolution of $\hbar k/18$. This method amounts to taking a snapshot of the velocity distribution of the Bloch states at time t_a in the accelerating optical potential.

Various momentum distributions of the atoms in the fundamental band as a function of time are shown in figure 3a, in the comoving frame. Under the influence of the external force, the initial momentum peak shifts linearly with time to the right while its weight decreases. Simultaneously a second peak emerges at a momentum separated by $-2\hbar k$; it becomes equal in weight to the first peak when $t_a = \tau_B/2$, where $\tau_B = \hbar k/ma$. It keeps growing until $t_a = \tau_B$, when the initial momentum distribution is recovered. The atoms have performed a full Bloch oscillation. Further evolution reproduces this pattern periodically. This figure directly illustrates the Bragg reflection of the matter wave when the atomic momentum p approaches the Bragg condition, $p = \hbar k$.

The mean atomic velocity of the atoms as a function of time is presented in figure 3b for a potential depth $U_0 = 2.3E_R$. The results clearly show the oscillatory motion of the particles, and the measured Bloch period (8.2 ms for an acceleration of 0.85 m/s^2) agrees with the calculated value to better than 1%. The corresponding oscillation of the mean position of the atoms has an amplitude of 2.3 μm and thus extends over 5.5 sites of the periodic potential, clearly defying classical laws. This coherent motion over several sites is also responsible for the pronounced asymmetry of the oscillation in figure 3b.

The fine control of the light potential allows precise control of the initial conditions of the atoms in the light field. For instance, atoms can be prepared with a particular quasi-momentum within any band ($n = 0, 1, 2,\ldots$). Bloch oscillations have thus been also observed in the first excited band ($n = 1$) and, by scanning over q, the Paris group have been able to measure the energies of the first two bands as a function of q.[8]

Ladders and tunnels

In the Austin experiments, a magneto-optical trap is also used to first trap and cool atoms, in this case sodium atoms. After the cooling and trapping stage, the trapping

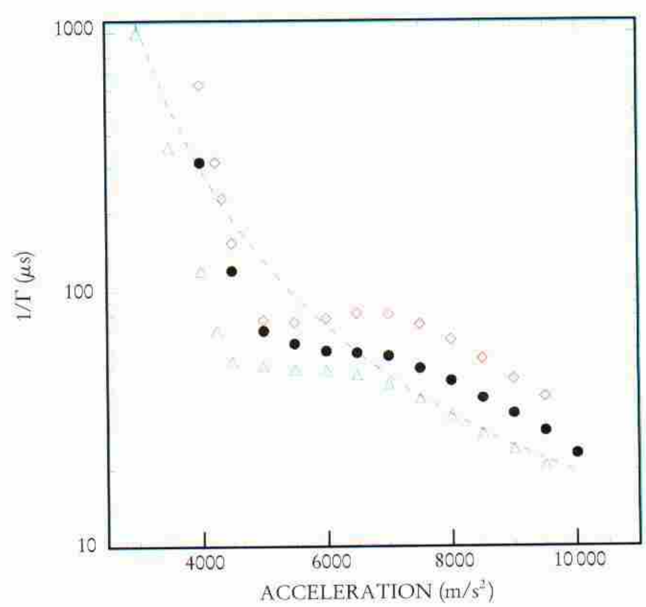


FIGURE 5. TUNNELING LIFETIME as a function of acceleration. The experimental data are marked by solid dots. The dashed line is the prediction of Landau–Zener theory. Theoretical quantum simulations (empty diamonds and triangles) use the experimental parameters within the experimental uncertainty, and bracket the observed lifetimes. Quantum interference effects cause the oscillation about the Landau–Zener curve. (From ref. 13.)

Particles	electrons	Cs atoms	Na atoms
Potential	superlattice	laser light	
Lattice constant	10 nm	426 nm	295 nm
Bloch bandwidth	1 meV	8 peV	100 peV
	250 GHz	2 kHz	25 kHz
Force	eE	ma	
Bloch period	0.4 ps	8 ms	25 μs
Amplitude of oscillations	several lattice constants		

beams and magnetic field gradient are turned off. When the far-detuned standing wave is turned on, about 10% of the atoms are then trapped in the lowest energy band. In the experiments described here, the standing wave is accelerated at rates of up to 1800 m/s^2, for interaction times of up to 1 ms. After interacting with the accelerating standing wave, the atoms drift freely in the dark for 3 ms, then the near-resonant trapping beams are turned back on without the magnetic field gradient, forming optical molasses. For short times, the atoms are essentially frozen in place in the molasses and a charge-coupled device camera records their fluoresence. The resulting two-dimensional images are integrated across the transverse direction to give the one-dimensional distribution along the standing-wave axis. This system was previously used to study quantum chaos in atom optics (see PHYSICS TODAY, June 1995, page 18).

To observe the Wannier–Stark ladder, a phase modulation of frequency ν_p is added to the accelerating optical potential. This AC field can drive transitions between the first two bands when it matches the transition frequency. For appropriate values of the acceleration, the Landau–Zener tunneling rate from the lowest band is negligible, while the tunneling rate from higher bands is large. Therefore only the atoms in the lowest band are accelerated, while atoms in the higher bands (such as those excited to the second band by the phase modulation) are left behind. Thus one can study the probability of excitation by applying a weak phase modulation and measuring the number of atoms that are accelerated, as shown in figure 1d. A theoretical analysis of this problem finds that the transition probability as a function of modulation frequency displays several equally spaced resonances, which are identified as an atomic Wannier–Stark ladder.[12]

A spectrum is measured by scanning ν_p, and figure 4 shows the result. The spectrum has two clear resonances (at $\nu_p \approx 85$ kHz and $\nu_p \approx 115$ kHz), which are necessary to determine the Wannier–Stark splitting. The theoretical curve is obtained by numerical integration of the time-dependent Schrödinger equation with parameters that match the experimental conditions. The observed splitting and lineshapes agree well with theory. One can also understand the spectrum as a quantum interference effect, and carry out a detailed study of the lineshapes. Repeating the experiment for various accelerations provides the Wannier–Stark splitting as a function of acceleration, and the results are consistent with the predicted linear scaling, within the experimental uncertainty.[7]

Both groups also have studied Landau–Zener tunneling in this system by measuring (in the absence of a phase modulation) the fraction of atoms that remain in the accelerating frame as a function of interaction time.[8,13] The survival probability, measured for a range of accelerations and well depths, was found to follow an exponential decay law. In the Austin experiments, tunneling was observed for accelerations in the range 4000–10 000 m/s^2, yielding lifetimes of 20–300 μs, as shown in figure 5. The oscillations around the Landau–Zener prediction (dashed line) are due to quantum interference effects that become more dominant at smaller values of the acceleration.

Future directions

To summarize, it is instructive to compare the parameters for the atomic physics and superlattice experiments (see the table above). These parameters span many orders of magnitude and illustrate the universal nature of quantum phenomena. Beyond the study of fundamental physics, the atomic physics experiments may have important applications to atom optics. The accelerating standing wave is an ideal method of launching a subrecoil sample of atoms, forming an ultracold atomic beam for atom optics and atomic interferometry. Several hundred $\hbar k$ of momentum can be imparted to the atoms in a coherent manner. One could use such an atom accelerator to launch a Bose condensate, forming a coherent and well-controlled beam of atoms analogous to the laser. (See PHYSICS TODAY, March 1997, page 17.) One could also make high-precision measurements of the photon recoil momentum and thus of h/m. Another interesting research direction is the effect of decoherence on quantum transport. One can introduce spontaneous scattering or noise in a controlled setting, and study the transition to classical behavior. More complicated beam configurations can also be used to study quantum transport in quasi-crystals.[14] It is clear that the study of atomic motion in optical lattices in this quantum regime should continue to provide new light and insight on quantum transport.

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