Spatial Nonlocal Pair Correlations in a Repulsive 1D Bose Gas

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(Received 31 October 2007; revised manuscript received 15 January 2008; published 25 April 2008)

We analytically calculate the spatial nonlocal pair correlation function for an interacting uniform 1D Bose gas at finite temperature and propose an experimental method to measure nonlocal correlations. Our results span six different physical realms, including the weakly and strongly interacting regimes. We show explicitly that the characteristic correlation lengths are given by one of four length scales: the thermal de Broglie wavelength, the mean interparticle separation, the healing length, or the phase coherence length. In all regimes, we identify the profound role of interactions and find that under certain conditions the pair correlation may develop a global maximum at a finite interparticle separation due to the competition between repulsive interactions and thermal effects.

DOI: 10.1103/PhysRevLett.100.160406 PACS numbers: 05.30.Jp, 03.75.Hh, 68.65.−k

The study of two-body and higher-order correlations is becoming an important theme in the physics of ultracold quantum gases. Correlation functions are observables that provide information about quantum many-body wave functions beyond the simple measurement of density profiles. They are of particular importance for the understanding of low dimensional and strongly correlated systems, atomic gases with exotic phases, and systems with multiple order parameters. Such correlations can manifest themselves in momentum space, as with pair correlations of a Fermi gas [1], and can be observed in time-of-flight experiments [2,3] which has been the standard method of measurement of degenerate gases. Nonlocal correlations in position, on the other hand, should ideally be performed *in situ* with a spatial resolution that is smaller than the typical correlation length, requiring spatially resolved *in-situ* single-atom counting [4–6].

In this Letter we address the problem of nonlocal two-particle correlations in a 1D Bose gas and propose an experimental method to measure them *in situ* using spatially resolved Raman transitions [7] and single-atom counting in an optical box trap [8]. We calculate the spatial second-order correlation function $g^{(2)}(r) = \langle\hat{\Psi}^\dagger(0)\hat{\Psi}^\dagger(r)\hat{\Psi}(r)\hat{\Psi}(0)\rangle/n^2$ for a uniform gas with repulsive δ-function interactions [9,10] (where $\hat{\Psi}$ is the field operator, r is the interparticle separation, and n is the 1D linear density) and assess the feasibility of the method with respect to the characteristic correlation lengths.

Given a particle at a certain location, the pair correlation $g^{(2)}(r)$ describes the probability of finding a second particle at a distance r compared to uncorrelated particles, and gives the characteristic length scale over which the density-density fluctuations decay. The knowledge of $g^{(2)}(r)$ in 1D Bose gases is of fundamental importance for the understanding of second-order coherence and for practical applications such as intensity interferometry in 1D environments. The recent experimental realizations of ultracold atomic gases in the 1D regime [11–15] and the fact that the 1D Bose gas problem is exactly integrable using the Bethe ansatz [9,10] make this an ideal system for investigating quantum many-body physics in previously unattainable regimes.

An early experimental measurement of atom-atom correlations using microchannel plate detectors was performed in an ultracold (but not degenerate) cloud of metastable neon [16]. More recently, the method was applied to quantum degenerate samples of helium atoms and to correlations resulting from condensate collisions [17]. Other experimental techniques to access higher-order correlations include shot-noise spectroscopy of absorption images [2,3,18], the measurement of three-body recombination and photoassociation rates [12,13,19], fluorescence imaging [4], and atom counting using high-finesse optical cavities [20].

Certain aspects of pair correlations in a repulsive 1D Bose gas have been studied previously, including the local correlation $g^{(2)}(0)$, asymptotic properties at large r, and the zero temperature behavior [21–29]. Here we extend these results to nonlocal correlations in six analytically tractable regimes of Refs. [25,26], ranging from strong to weak interactions—all at finite temperatures.

We begin by recalling that the second quantized Hamiltonian of the system is given by

$$\hat{H} = \frac{\hbar^2}{2m}\int dx\,\partial_x\hat{\Psi}^\dagger\partial_x\hat{\Psi} + \frac{g}{2}\int dx\,\hat{\Psi}^\dagger\hat{\Psi}^\dagger\hat{\Psi}\hat{\Psi}, \qquad (1)$$

where m is the mass and $g > 0$ is the coupling constant that can be expressed via the 3D s-wave scattering length a as $g \simeq 2\hbar^2 a/(ml_\perp^2) = 2\hbar\omega_\perp a$ [30]. Here, we have assumed that the atoms are transversely confined by a tight harmonic trap with frequency $\omega_\perp$ and that a is much smaller than the transverse harmonic oscillator length

$l_\perp = \sqrt{\hbar/m\omega_\perp}$. The 1D regime is realized when the excitation energy $\hbar\omega_\perp$ is much larger than the thermal energy T (with $k_B = 1$) and the chemical potential μ [26,31]. A uniform system in the thermodynamic limit is completely characterized [9,10] by two parameters: the dimensionless interaction strength $\gamma = mg/(\hbar^2 n)$ and the reduced temperature $\tau = T/T_d$ [25], where $T_d = \hbar^2 n^2/(2m)$ is the temperature of quantum degeneracy.

Although the uniform 1D Bose gas problem is exactly solvable by the Bethe ansatz [9], the cumbersome nature of the eigenstates restricts the straightforward calculation of correlation functions [22,32]. The Hellmann-Feynman theorem and the solutions to Lieb-Liniger [9] or Yang-Yang [10] integral equations—used for calculating the local correlation $g^{(2)}(0)$ [24–26]—can no longer be applied to $g^{(2)}(r)$ at arbitrary separation r. For sufficiently large r, the Luttinger liquid theory predicts universal features of $g^{(2)}(r)$ [33]. At $T = 0$ its behavior is characterized by interaction-dependent power-law approach to the uncorrelated value $g^{(2)}(r) = 1$ at $r \to \infty$. At finite T the approach to $g^{(2)}(r) = 1$ becomes exponential. The Luttinger picture is limited to temperatures smaller than the high energy cutoff of the order of T_d. To describe nonuniversal features of $g^{(2)}(r)$ at high temperatures and short distances it is necessary to adopt alternative theoretical techniques [24,25,34].

Strongly interacting regime [$\gamma \gg \max(1, \sqrt{\tau})$].—We employ perturbation theory with respect to the small parameter γ^{-1} [35] around the Tonks-Girardeau (TG) limit of impenetrable (hard-core) bosons [21]. At $T = 0$ we obtain [34] the known [22,29] result

$$g^{(2)}_{T=0}(r) = 1 - \frac{\sin^2(z)}{z^2} - \frac{4}{\gamma}\frac{\sin^2(z)}{z^2} - \frac{2\pi}{\gamma}\frac{\partial}{\partial z}\frac{\sin^2(z)}{z^2} + \frac{2}{\gamma}\frac{\partial}{\partial z}\left[\frac{\sin(z)}{z}\int_{-1}^{1} dt \sin(zt)\ln\frac{1+t}{1-t}\right], \quad (2)$$

where $z = \pi n r$. The last term here diverges logarithmically with z and can be regarded as a first-order perturbation correction to the fermionic inverse square power law. Accordingly, Eq. (2) is valid for $z \ll \exp(\gamma)$.

Well below quantum degeneracy, $\tau \ll 1$, finite temperature corrections are obtained using a Sommerfeld expansion around Fermi-Dirac distribution for quasimomenta at $T = 0$. For $rn \ll \tau^{-1}$ this gives [34] an additional contribution $\tau^2 \sin^2(\pi n r)/12\pi^2$ to the rhs of Eq. (2), which is negligible compared to the $T = 0$ result as $\tau \ll 1$. At $r = 0$, Eq. (2) gives perfect antibunching $g^{(2)}(0) = 0$, which corresponds to a fully "fermionized" 1D Bose gas, where the strong interatomic repulsion mimics the Pauli exclusion principle for intrinsic fermions. By extending the perturbation theory to include terms of order γ^{-2} we reproduce the known $T = 0$ result for the local correlation, $g^{(2)}(0) = 4\pi^2/3\gamma^2$ [24,25].

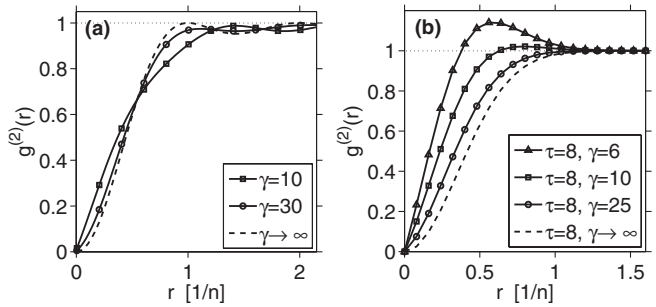

FIG. 1. Pair correlation $g^{(2)}(r)$ as a function of the relative distance r (in units of $1/n$) in the strongly interacting regime, $\gamma \gg 1$: (a) low-temperature TG regime, Eq. (2), for $\tau = 0.01$; (b) regime of high-temperature fermionization, Eq. (3).

In Fig. 1(a) we plot the function $g^{(2)}(r)$, Eq. (2), for various γ. The pair correlation exhibits oscillatory structure (similar to Friedel oscillations of a 1D electron gas with an impurity [36]), with the local maxima implying the existence of more likely separations between particles. Despite uniform density, this can be interpreted as a quasicrystalline order (with a period $\sim 1/n$) in the two-particle sector of the many-body wave function.

Well above quantum degeneracy, $\tau \gg 1$, we use perturbation theory combined with the Maxwell-Boltzmann distribution of quasimomenta. In this high-temperature "fermionization" regime [25] the characteristic momentum of the particles is $1/\Lambda_T$ and the perturbation parameter is $a_{1D}/\Lambda_T \ll 1$, where $a_{1D} = \hbar^2/mg \simeq l_\perp^2/a \sim 1/\gamma n$ is the 1D scattering length and $\Lambda_T = \sqrt{2\pi\hbar^2/mT}$ is the thermal de Broglie wavelength. This implies $\tau \ll \gamma^2$, and to first order in γ^{-1} we find [34]

$$g^{(2)}(r) = 1 - (1 - 2\tau n r/\gamma)e^{-\tau n^2 r^2/2}. \quad (3)$$

In the limit $r \to 0$ this leads to perfect antibunching, $g^{(2)}(0) = 0$, while the corrections [as in Ref. [25], $g^{(2)}(0) = 2\tau/\gamma^2$] are reproduced at second order in γ^{-1}. The correlation length associated with the Gaussian decay is given by $\Lambda_T = \sqrt{4\pi/(\tau n^2)}$. For not very large γ, the correlations show a nonmonotonic behavior with a global maximum at $r_{\max} \simeq \gamma/2\tau n$. This originates from the competition between the interaction induced repulsion at short range and thermal bunching [$g^{(2)}(r) > 1$] at $r \sim \Lambda_T$. As γ is increased the position of the maximum diverges and its value approaches 1 in a nonanalytical fashion $g^{(2)}(r_{\max}) \simeq 1 + (4\tau/\gamma^2)\exp(-\gamma^2/8\tau)$.

Figure 1(b) shows a plot of Eq. (3) for various γ and τ. For a well-pronounced global maximum, moderate values of γ^2/τ are required (such as $\gamma^2/\tau \simeq 5$, with $\tau = 8$, $\gamma = 6$), and these lie near the boundary of validity of our approximations ($\gamma^2/\tau \gg 1$). The exact numerical calculations of Ref. [27] provide further support for this result and show a similar maximum for $\gamma^2/\tau \simeq 0.25$ (with $\tau = 4\pi \times 10$ and $\gamma = 10$).

Weakly interacting regime [$\tau^2 \ll \gamma \ll 1$].—For weak interactions we rely on the fact that the equilibrium state of the gas is a quasicondensate with suppressed density fluctuations and a fluctuating phase [37,38]. The pair correlation function is close to 1 and the deviations can be calculated using Bogoliubov theory [24,25].

At sufficiently low temperatures, $\tau \ll \gamma \ll 1$, when vacuum fluctuations dominate the excitations and thermal fluctuations are a small correction we find [34]

$$g^{(2)}(r) = 1 - \sqrt{\gamma}[\mathbf{L}_{-1}(2\sqrt{\gamma}nr) - I_1(2\sqrt{\gamma}nr)] + \frac{1}{2\pi\sqrt{\gamma}n^2r^2} - \frac{\pi\tau^2}{8\gamma^{3/2}}\mathrm{cosech}^2\left(\frac{\tau n\pi r}{2\sqrt{\gamma}}\right), \quad (4)$$

where $\mathbf{L}_{-1}(x)$ is the modified Struve function and $I_1(x)$ is a Bessel function. The correlation length here is set by the healing length $\xi = \hbar/\sqrt{mgn} = 1/\sqrt{\gamma}n$. For $r \gg \xi$ and finite τ, the last term in Eq. (4) dominates the others and gives an exponential decay to the uncorrelated value of $g^{(2)}(r) = 1$ (for $\tau \to 0$ one has a power-law decay). Even at $T = 0$, oscillating terms are absent, in contrast to the strongly interacting regime, Eq. (2). The limit $r \to 0$ reproduces the result of Eq. (9) of Ref. [25], $g^{(2)}(0) = 1 - 2\sqrt{\gamma}/\pi + \pi\tau^2/(24\gamma^{3/2})$. In Fig. 2(a) we plot Eq. (4) for different values of γ, and we note that the finite temperature correction term is negligible here.

In the opposite limit, dominated by thermal fluctuations corresponding to $\gamma \ll \tau \ll \sqrt{\gamma}$, we find [34]

$$g^{(2)}(r) = 1 + \frac{\tau}{2\sqrt{\gamma}}e^{-2\sqrt{\gamma}nr} - \sqrt{\gamma}[\mathbf{L}_{-1}(2\sqrt{\gamma}nr) - I_1(2\sqrt{\gamma}nr)], \quad (5)$$

valid for $r/\xi \lesssim 1$. The last two terms are due to vacuum fluctuations and are a negligible correction, so the leading term gives an exponential decay [see Fig. 2(b)] with a correlation length given by the healing length ξ. The peak value is $g^{(2)}(0) = 1 + \tau/(2\sqrt{\gamma})$, in agreement with Ref. [25]. For $r/\xi \gg 1$ vacuum fluctuations dominate and we reproduce the asymptotic behavior of Eq. (4).

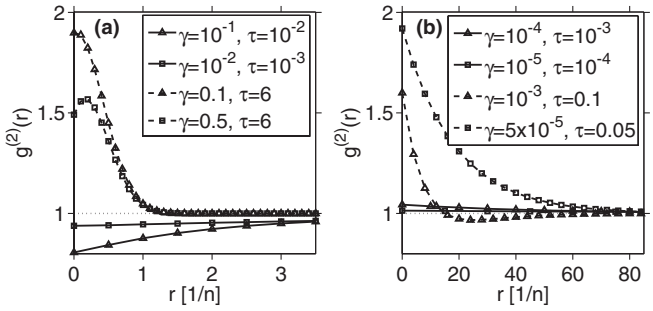

FIG. 2. Pair correlation $g^{(2)}(r)$ in the weakly interacting and nearly ideal-gas regimes. (a) Solid lines—low-temperature weakly interacting gas at $\tau \ll \gamma \ll 1$, Eq. (4); dashed lines—DC regime, Eq. (7). (b) Solid lines—weakly interacting gas at $\gamma \ll \tau \ll \sqrt{\gamma}$, Eq. (5); dashed lines—DQ regime, Eq. (9).

Nearly ideal-gas regime [$\gamma \ll \min\{\tau^2, \sqrt{\tau}\}$].—Finally, we present the results for the decoherent regime, where both the density and phase fluctuations are large and the local pair correlation is close to the result for noninteracting bosons, $g^{(2)}(0) = 2$. Depending on the temperature τ, we further distinguish two subregimes: decoherent quantum (DQ) regime for $\tau \ll 1$, and decoherent classical (DC) regime for $\tau \gg 1$. Both can be treated using perturbation theory with respect to the coupling constant g around the ideal Bose gas result of Ref. [18].

In the DQ regime, with $\sqrt{\gamma} \ll \tau \ll 1$ [25], the nonlocal pair correlation is [34]

$$g^{(2)}(r) = 1 + [1 - 4\gamma(1 + \tau nr)/\tau^2]e^{-\tau nr}, \quad (9)$$

with the peak value $g^{(2)}(0) = 2 - 4\gamma/\tau^2$ [25]. For $\gamma = 0$ the correlations decay exponentially with the characteristic correlation length which coincides with the phase coherence length $l_\phi \simeq \hbar^2 n/mT = 2/\tau n$ [25] and is responsible for the long-wavelength phase fluctuations. For $\gamma > 0$, $g^{(2)}(r)$ becomes nonmonotonic with a minimum at $nr_{\min} = \tau/4\gamma \gg 1$ before reaching its uncorrelated value $g^{(2)}(r \to \infty) = 1$. Thus, at intermediate range we have weak antibunching due to interatomic repulsion, while at short range we have typical ideal-gas bunching due to exchange interaction, as shown in Fig. 2(b).

In the DC regime ($\tau \gg \max\{1, \gamma^2\}$), the pair correlation is given by [34]

$$g^{(2)}(r) = 1 + e^{-\tau n^2 r^2/2} - \gamma\sqrt{2\pi/\tau}\,\mathrm{erfc}(\sqrt{\tau n^2 r^2/2}), \quad (7)$$

where $\mathrm{erfc}(x)$ is the complimentary error function. At $r = 0$ we have $g^{(2)}(0) = 2 - \gamma\sqrt{2\pi/\tau}$ [25] as $\mathrm{erfc}(0) = 1$. In the noninteracting limit ($\gamma = 0$) we recover the well-known result for the classical ideal gas [39] characterized by Gaussian decay with a correlation length Λ_T. For $\gamma > 0$ we observe [see Fig. 2(a)] the emergence of nonmonotonic behavior, with a global maximum $g^{(2)}(r_{\max}) = g^{(2)}(0) + 2\gamma^2/\tau$ at nonzero separation $nr_{\max} = 2\gamma/\tau \ll 1$. As γ is increased, there is a continuous transition from the DC regime to the regime of high-temperature fermionization, with $g^{(2)}(0)$ reducing further and the maximum moving to larger distances.

We now discuss experimental methods to measure pair correlations. Local correlations have been measured by photoassociation [13] and by three-body loss [12,19]; however, nonlocal spatial correlations have not been measured *in situ* to the best of our knowledge. We discuss one possible implementation of spatially resolved imaging in the context of a 1D gas. The method is closely related to the spatially resolved measurement of Ref. [7]. The first step is to "freeze-in" the correlations by turning on a deep standing wave of far-detuned light, which breaks up the distribution into discrete packets, separated by a half-

wavelength. The next step is to apply a magnetic field that varies linearly in magnitude, creating a spatially dependent Zeeman shift. Next, a stimulated Raman transition is driven such that atoms in two sites are transferred to a different hyperfine state. This can be accomplished by using two frequencies on the Raman beam corresponding to two different locations. The unaffected atoms can be removed with a pulse of resonant light and the remaining atoms counted.

For a specific example, consider a degenerate gas of sodium atoms in an optical box trap [8], in the $F = 2$, $m_F = -2$ state. After freeze-in with a $\lambda = 532$ nm lattice, we apply a magnetic field gradient of 150 G/cm. We then drive a two-photon stimulated Raman transition to the $F = 1$, $m_F = -1$ state with Raman beams at 532 nm with optical power of 2.5 mW focused to 50 μm and a pulse duration of 260 μs. A detuning of 53 kHz corresponds to a shift of one lattice site and therefore a resolution of $\sim$266 nm. For typical 1D gas parameters, this resolution is sufficient to resolve the characteristic correlation lengths found here. A related simpler method, but with somewhat lower resolution, has been recently proposed in Ref. [40].

In summary, we have calculated nonlocal pair correlations in a uniform 1D Bose gas in six physically relevant regimes. The correlations can be measured using spatially resolved single-atom counting. We have shown explicitly that the characteristic global correlation lengths are given by one of four length scales: the thermal de Broglie wavelength, the mean interparticle separation, the healing length, or the phase coherence length. In all cases we identified the profound role of interactions that can lead to nontrivial structures with local maxima or minima at a finite interparticle separation.

A. G. S., M. J. D., and K. V. K. acknowledge stimulating discussions with P. Deuar, P. Drummond, A. Cherny, and J. Brand, and the support by the Australian Research Council. D. M. G. acknowledges support by EPSRC grant No. EP/D072514/1. K. V. K. and D. M. G. acknowledge the hospitality and support of the Institut Henri Poincaré. M. G. R. acknowledges support from the R. A. Welch Foundation, The National Science Foundation, and the Sid W. Richardson Foundation.

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