Quantum Tweezer for Atoms

Roberto B. Diener,[1] Biao Wu,[1,3] Mark G. Raizen,[1,2] and Qian Niu[1]
[1]*Department of Physics, The University of Texas, Austin, Texas 78712-1081*
[2]*Center for Nonlinear Dynamics, The University of Texas, Austin, Texas 78712-1081*
[3]*Solid State Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6032*
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We propose a quantum tweezer for extracting a desired number of neutral atoms from a reservoir. A trapped Bose-Einstein condensate is used as the reservoir, taking advantage of its coherent nature, which can guarantee a constant outcome. The tweezer is an attractive quantum dot, which may be generated by red-detuned laser light. By moving at certain speeds, the dot can extract a desired number of atoms from the condensate through Landau-Zener tunneling. The feasibility of our quantum tweezer is demonstrated through realistic and extensive model calculations.

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The manipulation and control of isolated single neutral atoms has been a long term goal with important applications in quantum computing [1,2] and fundamental physics. Trapping and cooling of single neutral atoms was first achieved in magneto-optical traps and more recently in a dipole trap [3–6]. Despite these impressive successes, all existing methods share a common weakness: The trapping process itself is random and not deterministic. In this Letter, we propose a quantum tweezer that can extract a definite number of atoms from a reservoir at will, with the atoms in the ground state of the tweezer. A trapped Bose-Einstein condensate (BEC) is used as a reservoir, and its coherent nature makes the constancy of the output possible. An attractive quantum dot, created by a focused beam of red-detuned laser light, serves as a quantum tweezer to extract a desired number of atoms from the BEC reservoir.

In a typical operation of the quantum tweezer, a quantum dot is turned on adiabatically inside the bulk of the BEC and moves out of the BEC at a certain speed so that a desired number of atoms is extracted (see Fig. 1). In the initial stage of this operation, it is important that the system remains in the ground state of the trap + dot potential. The superfluidity of the BEC helps to suppress the excitations which might otherwise be induced by the turning on and movement of the quantum dot. The speed of the dot just needs to be slower than the speed of sound, and the rate of turning on of the dot potential should be smaller than the frequency of phonons whose wavelength is comparable to the size of the dot.

The crucial part of the tweezer operation is when the dot moves out of the BEC. Inside the BEC, when the coupling between the trap and the dot is still stronger than the atom self-interaction within the dot, the system is in a coherent state in which the number of atoms in the dot strongly fluctuates. Outside the BEC, the coupling drops exponentially with distance and eventually becomes negligible compared to the self-interaction; the eigenstates of the system are then Fock states in which the dot contains a definite number of atoms. In the general case, the dot exits the condensate in a superposition of eigenstates. However, under certain circumstances, we can steer the state into a prescribed final state, with a definite number of particles in the dot.

Starting from the ground state of the system with the dot at a certain position inside the BEC, we start moving the dot outwards. At an infinitesimally slow speed, the system always stays in the lowest energy state and no atoms are extracted, simply because moving out of the BEC costs potential energy of the atoms. At some finite speed, the system may get stuck in a nonzero number state of the dot, and become decoupled from the BEC before the atoms in the dot have a chance to leak back. In the following, we

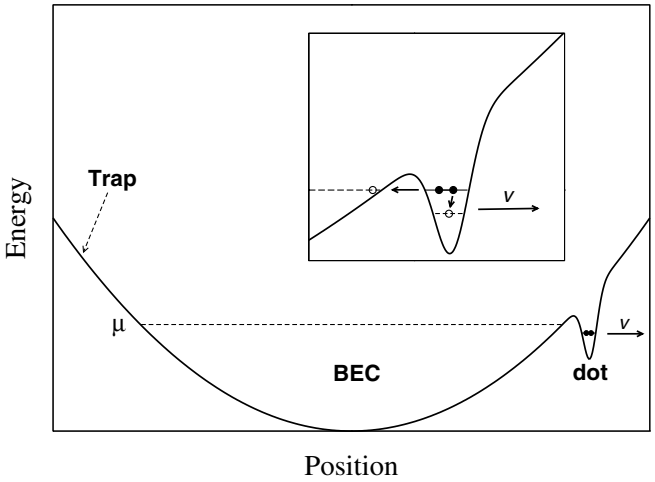

FIG. 1. A quantum dot (tweezer) moves out of a trapped BEC (reservoir) with the speed of v. The inset illustrates that a resonance occurs as the dot moves further away from the trap center such that the energy of the atoms matches the chemical potential μ of the condensate. If one of the atoms is tunneled into the BEC, the energy level of the dot is lowered, due to the absence of repulsion from the lost atom. Thus, no other atom has a chance of leaking back to the condensate at this position.

will give a detailed account of this phenomenon through a realistic model calculation. In Fig. 2, we show a result for the probability of extracting a single atom as a function of the speed of the dot. The plateau extends several orders of magnitude of the speed, demonstrating the robustness of our quantum tweezer.

Our focus is on the crucial stage of the quantum tweezer operation—when the dot is leaving the BEC cloud. In this case, the density is low and the interaction between atoms in the dot and atoms in the condensate is weak. The state of the system can then be expressed as a combination of atoms in the dot (with wave function ϕ_d) and atoms in the BEC trap (with wave function ϕ_B, properly orthogonalized to ϕ_d [7]). These two wave functions are chosen as the adiabatic ground state of the system when the dot is motionless and the coupling between these two sets of atoms is negligible.

The Hamiltonian of the system is $\hat{H} = \int dx\, \hat{\Psi}^\dagger(x) \times [-\frac{\hbar^2}{2M}\nabla^2 + V_t(x) + V_d(x,t) + \frac{g}{2}\hat{\Psi}^\dagger(x)\hat{\Psi}(x)]\hat{\Psi}(x)$. We can write $\hat{\Psi}(x) = \phi_B(x)\hat{c} + \phi_d(x)\hat{a}$ in the weak coupling limit, where $\hat{c}$ annihilates an atom in the trap and $\hat{a}$ annihilates an atom in the dot. We shall denote the state with n atoms in the dot (and $N - n$ atoms in the BEC) by $|n\rangle$; an atom jumping from the dot to the BEC corresponds to the transition $|n\rangle \rightarrow |n-1\rangle$. Given that $\phi_d(x)$ is much more localized than $\phi_B(x)$, the repulsion felt by the atoms in the dot is stronger than the one felt by the ones in the BEC. This asymmetry between the two potentials yields n much smaller than N, in general, and sets our system apart from two-state condensates discussed elsewhere [8].

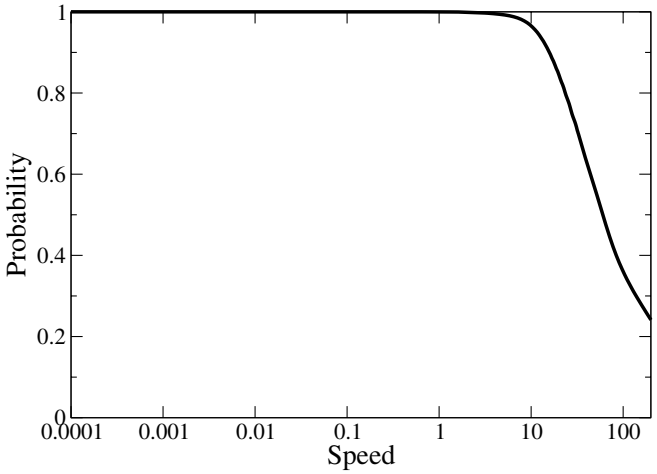

FIG. 2. The probability of extracting a single atom as a function of the speed of the dot. The calculation was performed for a one-dimensional BEC with $N = 10\,000$ atoms in a harmonic trap with frequency $\omega = 0.005$. The dot is a square well with depth $U_0 = 8$ and width $a = 1$, and the effective coupling constant is $g = 8$. The units for all these parameters are defined in the text. For sodium, speed is measured in units of 2.75 mm/s, so that for many speeds shown it takes a fraction of a second to extract one atom. The plateau exhibited extends several orders of magnitude.

The nonvanishing matrix elements of the Hamiltonian are (for $n \ll N$)

$$\langle n|\hat{H}|n\rangle = E_n \equiv nE_1 + \frac{n(n-1)}{2}\nu, \quad (1)$$

$$\langle n|\hat{H}|n+1\rangle = \langle n+1|\hat{H}|n\rangle = \sqrt{n+1}[\Delta + nG], \quad (2)$$

$$\langle n|\hat{H}|n+2\rangle = \langle n+2|\hat{H}|n\rangle = \sqrt{(n+1)(n+2)}A. \quad (3)$$

The parameters depend on the position x_d of the dot and can be explicitly calculated. $E_1 = \epsilon_d + V_t(x_d) - \mu + 4A$ accounts for the energy difference between the ground state in the dot and the chemical potential μ, while $\nu = gJ_{0,4}$ represents the repulsion an atom in the dot feels from another atom there. We have defined the generalized overlap integrals as $J_{m,n} = \int dx(\phi_B)^m(\phi_d)^n$. Notice that E_1 increases as the dot moves away from the center of the BEC. The off-diagonal terms are the couplings that allow an atom to tunnel from the dot to the BEC (or vice versa) either by itself or in pairs. The two terms in $\Delta = \sqrt{N}[\langle\phi_B|V_t|\phi_d\rangle + gNJ_{3,1}]$ correspond to quantum tunneling over a barrier and the interaction of a particle in the dot with three atoms in the BEC trap, respectively: this last term dominates when the dot is inside the BEC cloud. Equivalently, $G = g\sqrt{N}J_{1,3}$ is due to the interaction of three atoms in the dot with one atom in the trap. Finally, $A = gNJ_{2,2}/2$ is due to the interaction of two atoms in the trap with two in the dot. Outside of the BEC, the off-diagonal terms vanish exponentially, since the overlap integrals do so.

Although our scheme works in any dimensionality, we concentrate in what follows on a dilute, one-dimensional condensate [9] as an example. Such a system can be obtained by tightly confining the cloud in the transverse directions, in which the atomic dynamics are frozen out. The coupling constant is $g = 4\pi a_s\hbar/(M_{\text{atom}}L^2)$, where a_s is the s-wave scattering length and L is the length of the perpendicular confinement. We shall express our results in the following units: length in units of $L_0 = 1\ \mu$m, time in units of $M_{\text{atom}}L_0^2/\hbar$, and energy in units of $\hbar^2/(M_{\text{atom}}L_0^2)$.

We plot in the bottom panel of Fig. 3 our calculation of the energy levels as a function of the position for a harmonic trap with frequency $\omega = 0.005$ and $N = 10\,000$ atoms. The dot used is a square potential with depth $U_0 = 8$ and width $a = 1$; the coupling constant is $g = 8$. The edge of the condensate cloud is marked by the dotted line [10]. For comparison, the top panel shows the curves $E_n(x)$, corresponding to the energies of states with n atoms in the dot in the absence of the tunneling terms. The wave function for the BEC was calculated by numerical solution of the Schrödinger equation in imaginary time [11].

Let us consider the evolution of the number of atoms in the dot as the dot moves out of the BEC, with the help of Fig. 3. It is possible for an atom to tunnel out of the dot when there is no extra energy required to do so, i.e., when the energy for n atoms in the dot is equal to the energy of $n-1$ atoms in the dot. This is shown in the top panel of Fig. 3 as the locations where the curves for E_n and E_{n-1}

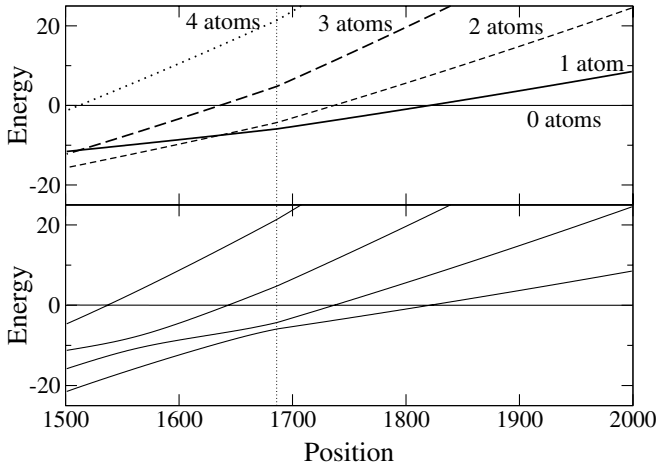

FIG. 3. Energy levels as a function of position for different numbers of atoms in the dot (bottom panel). In the top panel, the energy levels E_n (ignoring the off-diagonal terms) are plotted for comparison. The dotted line represents the edge of the condensate. The parameters used are the ones of Fig. 2.

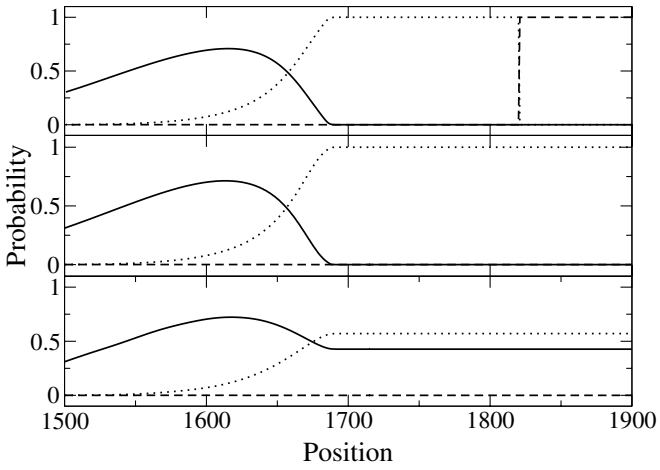

FIG. 4. Probability of finding no atoms (dashed), one atom (dotted), and two atoms (solid line) inside the dot as a function of position for three different speeds. The top panel is the adiabatic result ($v \to 0$), the middle panel is for $v = 1$, and the bottom panel for $v = 50$.

cross. These crossings also correspond to the resonance condition that we see in Fig. 1, the extra energy due to the nth atom being equal to the chemical potential μ of the condensate [12]. The possibility of tunneling out is realized by the off-diagonal terms, which open up energy gaps in the crossings as seen in the bottom panel of Fig. 3. As demanded by the quantum adiabatic theorem, starting in the ground state (the lowest curve) at some position x, if the dot moves infinitesimally slow the system remains in its ground state by losing one atom at each avoided crossing. When the dot is finally outside the condensate, no more atoms are left in it. Note that only one atom is allowed to leak out of the dot at each crossing with the ground state, due to the diminishing repulsion between atoms in the dot as there are less atoms in it (see Fig. 1). The next atom would have a chance of leaking to the condensate as the dot moves further away from the center of the BEC trap and lifts up its potential energy.

On the other hand, if the dot is moving at a finite speed there is a probability for the system to tunnel through the gap into an excited state, which corresponds to an atom not leaking back to the condensate when it is energetically allowed to do so. In the extreme (sudden) case in which the dot moves at infinite speed, the system remains in its initial state, the atoms in the dot having no time to leak. For an atom moving at speed v, the probability for Landau-Zener (LZ) tunneling [13] depends on the width δ of the gap as $P_{\mathrm{LZ}} = \exp(-\delta^2/2\alpha v)$, where α is the difference in the slopes of the two intersecting curves, which is approximately equal to dE_1/dx. For a dot moving at fixed speed v, the evolution is adiabatic ($P_{\mathrm{LZ}} < 0.01$) if $\delta > (9.21\,\alpha v)^{1/2}$ and sudden ($P_{\mathrm{LZ}} > 0.99$) if $\delta < (0.02\,\alpha v)^{1/2}$.

We can rewrite the resonance condition as $E_1 = -(n-1)\nu$. Using the definition of E_1 and the fact that outside the BEC the off-diagonal terms are exponentially small, we can see that transitions take place outside the BEC [i.e., $V(x_d) - \mu > 0$] when

$$\epsilon_d + (n-1)\nu < 0. \tag{4}$$

Notice, in particular, that the $|1\rangle \to |0\rangle$ transition always takes place outside the BEC. Such transitions are typically sudden in the sense of the LZ tunneling, due to the smallness of δ there. We can design a situation in which all transitions with $n \geq n_0$ take place inside the condensate while those with $n < n_0$ occur outside the cloud. A dot moving at a speed slow enough for all transitions inside the cloud to be adiabatic extracts then exactly n_0 atoms.

This is demonstrated in Fig. 2, where we show the probability of extracting one atom as a function of the

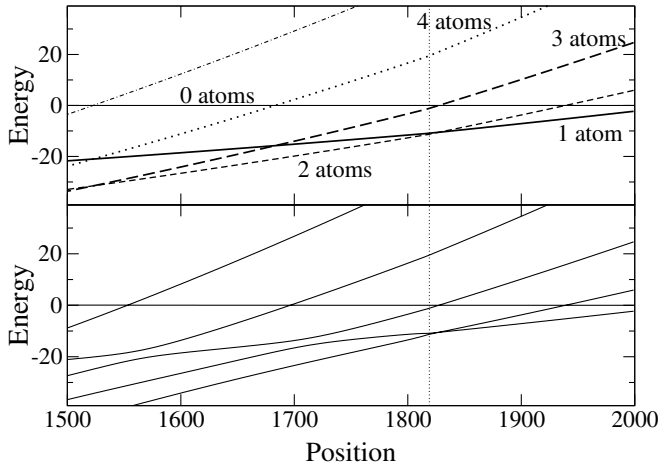

FIG. 5. Energy levels as a function of position (bottom panel) for a one-dimensional condensate with $\omega = 0.005$, $N = 10\,000$ atoms, potential depth $U_0 = 13.5$, and width $a = 1$, and the effective coupling constant is $g = 10$. The dotted line represents the edge of the condensate. The average energy E_n of states $|n\rangle$ are plotted in the top panel for comparison.

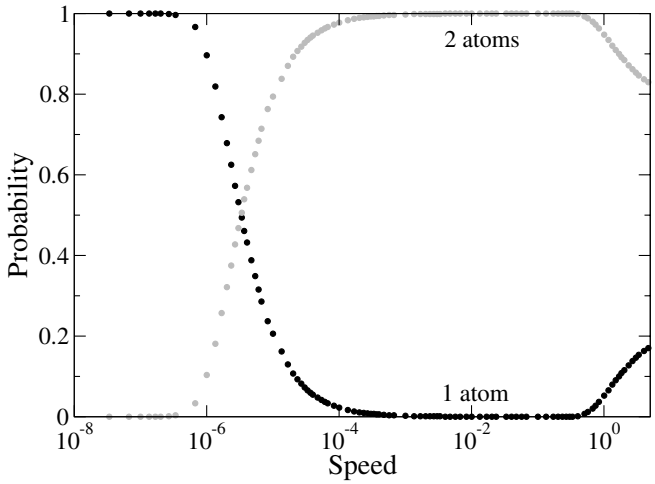

FIG. 6. The probability of extracting one (black) and two atoms (gray) as a function of speed for the case described in Fig. 5. In this case, we can see two different plateaus, showing that the quantized output depends strongly on the speed of the dot.

velocity of the dot. The calculation was performed assuming the system to be initially in the ground state some distance inside the BEC and integrating the equations of motion with the Hamiltonian matrix (1-3).

In Fig. 4, we show the calculated evolution of the probability of finding no atoms (dashed), one atom (dotted), and two atoms (solid lines) in the dot as a function of position for three different speeds of the dot. The top panel shows the adiabatic result, obtained for v, that is several orders of magnitude smaller than the smallest speed in Fig. 2. As expected, after the $|2\rangle \rightarrow |1\rangle$ transition has taken place there is one atom in the dot, while after the $|1\rangle \rightarrow |0\rangle$ transition there are none left. The middle panel corresponds to a slow speed, for which the evolution is adiabatic everywhere except at the $|1\rangle \rightarrow |0\rangle$ transition, whose LZ tunneling is sudden. Under these circumstances, the dot ends up with exactly one atom once it is outside the condensate. The last panel shows the evolution for an even larger speed. In this case, the LZ tunneling takes place only partially and the outcome corresponds to a superposition of number states. One important point illustrated clearly in this figure is that there is no definite number of atoms in the dot during most of the evolution.

We can also find a situation in which for a certain range of speeds the output is two particles, while for a different range (and for all other parameters fixed) the output is one particle, both with high certainty. This is achieved by choosing the $|2\rangle \rightarrow |1\rangle$ transition next to the edge of the cloud, so that both it and the $|3\rangle \rightarrow |2\rangle$ transitions have appreciable gaps opened at the crossing. In Fig. 5, we show the energy levels for this case, while in Fig. 6 we see the probability of extracting one and two atoms as a function of the speed of the dot. We can clearly see that there are two separate plateaus at different ranges of speed.

Finally, we make some remarks about the generality of our method. The parameters defining the Hamiltonian matrix elements depend on the density of the trapped BEC at the location of the dot. We expect the general behavior found in our one-dimensional calculations to remain true in any number of dimensions, since the motion of the dot singles out a direction and the other dimensions get effectively integrated out. Moreover, since the density of the BEC at the location of the dot is unchanged by the location of other dots elsewhere, we can consider a train of dots extracting atoms from the BEC independently of each other.

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