Recovery of classically chaotic behavior in a noise-driven quantum system

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The quantum kicked rotor is studied in a regime of high amplitude noise. A transition to diffusive behavior is observed as dynamical localization, characterized by suppressed diffusion and exponential momentum distributions, is completely destroyed by noise. With increasing noise amplitude, further transition to classical behavior is shown through an accurate quantitative analysis, which demonstrates that both the energy growth and the momentum distributions are reaching their classical limits. The importance of short-time correlations in the recovery of classically chaotic behavior is discussed.

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The relation between chaotic classical systems and their quantum counterparts is an important testing ground for the correspondence principle of quantum mechanics. One of the most important fingerprints of quantum behavior is the existence of quantum coherences. In the context of chaos, quantum coherences are known to result in dynamical localization, which is the suppression of classical diffusion due to the localization of quasienergy states [1]. It is now widely accepted that the destruction of quantum interference, known as decoherence, in a macroscopic classical world is provided by the coupling of a quantum system to its environment [2,3]. The sensitivity of quantum coherences to such coupling, in particular to noise and dissipation, has been the focus of much theoretical work [4–6], where the simplest prototype example for the suppression of classical chaos, the quantum δ-kicked rotor, has been studied. It has been found that the quantum coherences are extremely fragile in the semiclassical limit (small $\hbar$) where any amount of noise, however small, restores classical diffusion [4]. In the quantum regime, however, the system is not as vulnerable, and only strong noise has been claimed to break quantum interference completely [5].

Experiments on noise-induced decoherence have been carried out in different types of systems. The transition from subdiffusive to diffusive (though not necessarily classical) behavior has been studied in Rydberg atoms [7], but no direct comparison to the classical picture has been made. A very close quantum-classical correspondence has been observed in the experiments on the ionization of hydrogen atoms [8], although some of the initial states appeared to be very robust to the applied noise and did not follow the classical model. Atom optics offers many new tools for exploring the field of quantum chaos [9,10]. Decoherence in this system leads to the destruction of dynamical localization resulting in a subsequent diffusive growth in momentum, as has been experimentally demonstrated [11,12]. However, even though the qualitative signature of decoherence was obvious, the question of whether the classical limit could be reached remained unclear. In this paper we show experimentally, and prove through an accurate quantitative analysis, that amplitude noise may not only destroy dynamical localization, but may also drive the quantum system to the classical limit.

The atom-optics realization of the quantum kicked rotor may be described as a two-level atom (with transition frequency ω_0) interacting with a pulsed standing wave of near-resonant light of frequency ω_L. For sufficiently large detuning $\Delta_L = \omega_L - \omega_0$, the excited state amplitude can be adiabatically eliminated [13], and the atom can be treated as a point particle. In this approximation, the center-of-mass motion of the atom is described by the Hamiltonian

$$H = \frac{p^2}{2M} + V_0 \cos(2k_L x) \sum_{n=1}^{N} F(t - nT), \tag{1}$$

where the kick strength is determined by the dipole potential $V_0 = \hbar\Omega^2/8\Delta_L$, k_L is the wave number, T is the pulse period, Ω is the resonant Rabi frequency, and $F(t)$ is a pulse shape function of unit height and duration $t_p \ll T$. In scaled units, the Hamiltonian (1) acquires the familiar dimensionless form, which in the limit of arbitrarily short pulses describes the dynamics of the δ-kicked rotor after N kicks [14]. Two parameters are of particular importance. First, in the quantized model, the scaled conjugate variables p' and x' satisfy the commutation relation $[x', p'] = i\bar{k}$, where $\bar{k} = 4\hbar k_L^2 T/M$ is a scaled Planck constant, which is the measure of ‘‘classicality’’ of the system. Second, the stochasticity parameter, represented by the scaled kick strength, determines the classical phase space structure. It can be written as $K = (\bar{k}/\hbar)\eta V_0 T$, where we have defined the integrated pulse area $\eta = (1/T)\int_{-\infty}^{\infty} F(t)dt \propto t_p/T$. Note, however, that due to the finite pulse width, K is effectively reduced with increasing momentum. For $K > 4$ (which applies for our experimental conditions), the classical phase space is predominantly chaotic, and the classical dynamics exhibit approximately diffusive energy growth.

We may now introduce amplitude noise by replacing the fixed kick amplitude K with a random, step-dependent amplitude $(K + \delta K_n)$, where δK_n is a random deviation for the nth kick, uniformly distributed between $-\delta K_{\text{p-p}}/2$ and $+\delta K_{\text{p-p}}/2$. In the remainder of this paper we will refer to the quantity $(\delta K_{\text{p-p}}/K)$ (the ratio of peak-to-peak deviation to the mean kick amplitude) as the amount of amplitude noise. According to a simple heuristic picture of delocalization [4,5], amplitude noise causes transitions between the quasienergy eigenstates on a time scale shorter than the

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quantum break time. As a result, quantum coherences are destroyed more quickly than the dynamical localization is established, and the system should exhibit classical diffusion. Our experiments show that for reaching the classical limit, the quantum correlations must be destroyed on all time scales (including short-time kick-to-kick correlations), which requires strong amplitude noise. Hereafter, by strong noise we mean $\delta K_{\text{p-p}}$ of the order K. Note that classical diffusion is also susceptible to the applied noise and therefore the classical diffusion coefficient will be modified by the noise.

To study the effects of noise, we used the same experimental setup as in our earlier quantum chaos experiments [11,14,15]. Typically, 10^6 cesium atoms are trapped and cooled in a standard six-beam magneto-optical trap, and then optically pumped to the $(6S_{1/2},F=4)$ hyperfine state. The initial spatial and momentum distribution widths are $\sigma_x = 0.15$ mm and $\sigma_p = 8\hbar k_{\text{L}}$, respectively. After turning the trapping fields off, the atoms are ''kicked'' by the pulsed standing wave produced by a stabilized single-mode Ti:sapphire laser. The laser beam is linearly polarized, spatially filtered, aligned with the atoms, and retroreflected through the chamber to form a standing wave. The beam has a typical power of 470 mW at the chamber and a waist of 1.5 mm. For all of the experiments described here, the detuning of the interaction beam is 6.1 GHz to the red of the $(6S_{1/2},F=4) \rightarrow (6P_{3/2},F=5)$ cycling transition, with typical drifts of about 100 MHz. The pulse sequence is produced by an acousto-optic modulator, and consists of a series of 295 ns [full width at half maximum (FWHM)] pulses with a rise/fall time of 70 ns and less than a 3 ns variation in the pulse duration. The pulse period is $T=20$ μs, which corresponds to $\bar{k}=2.08$. The detection of momentum is accomplished by letting the atoms drift in the dark for a controlled duration (15 ms). The trapping beams are then turned on in zero magnetic field, forming an optical molasses that freezes the position of the atoms [9]. The atomic position is recorded via fluorescence imaging in a short (10 ms) exposure on a cooled charge-coupled device (CCD) camera. For the known free-drift time, the final spatial distribution determines the one-dimensional momentum distribution along the direction of the interaction beam, from which the total kinetic energy of the atoms is calculated.

Our first goal is to compare the experimental results with the classically expected diffusion for a given amount of amplitude noise. In Fig. 1 the measured energies are plotted versus the number of kicks for different noise levels. In the case of no noise, dynamical localization is manifested by suppressed energy growth. As the noise is increased, diffusion also increases due to the destruction of quantum coherences, and a clear trend towards the classically expected linear energy growth is observed. In order to facilitate an accurate comparison of the experimental data to the classical limit, we calculate the behavior of the classical kicked rotor using Monte Carlo simulations, where we take into account several systematic effects that are present in our measurements. First, the nonzero temporal width of the standing wave pulses leads to an effective reduction of the kick strength at higher momenta. This effect is sensitive to the temporal profile of the pulse, $F(t)$. In the simulations, the classical equations of motion are integrated using an analytical formula that accurately fits the measured pulse shape.

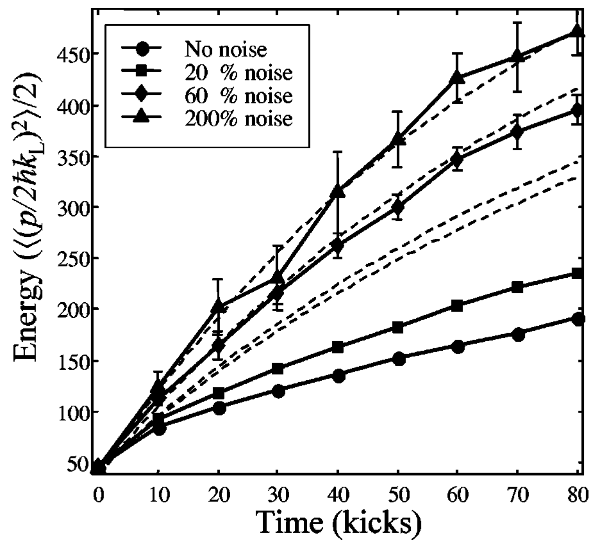

FIG. 1. Energy versus time for increasing levels of amplitude noise. The dots, connected by solid lines, represent experimental results. The error bars of one standard deviation show only statistical variations among ten different experimental realizations of each level of noise, and do not include systematic effects described in the text. Dashed lines are the classical simulations with the systematic effects taken into account (higher energy curves correspond to higher noise levels), for the measured value of $K=11.2$.

The finite spatial size of the interaction beam is also important, since the atoms are moving transversely during the free evolution between the kicks, which results in variations of the kick strength over the atomic distribution. This effect is taken into account in the calculations by introducing a position-dependent kick strength for each step of the interaction according to the transverse position of each particle and the measured spatial profile of the beam. Finally, the calculated distributions are corrected for the intensity profile of the freezing molasses beams which effectively decreases the wings of the measured momentum distributions. All calculations presented here have been performed with experimentally determined values of K, noise levels, and numbers of kicks, with no additional fitting parameters. The systematic uncertainty in K is $\pm 10\%$, and is mostly due to the measurement of the laser beam profile and absolute power.

The results of the classical simulations are shown as dashed lines in Fig. 1. Due to the noise and the above-mentioned systematic effects, the classical curves deviate from straight diffusive lines. While the calculations are unable to explain the system behavior for small noise amplitudes (corresponding to incomplete destruction of localization), they are in good agreement with the measured energy values in the strong noise limit. From this analysis we may conclude that the robustness of the quantum system to the applied perturbation is relatively high.

The unsuppressed energy growth represents a necessary, though not sufficient, condition for reaching the classical limit. An important hallmark of classicality is the Gaussian momentum distributions as opposed to the exponentially localized line shapes [9,10]. To analyze the effect of noise on the momentum distributions, we compare experimentally observed distributions with the results of classical simulations. In each plot of Fig. 2, the time evolution of the system is

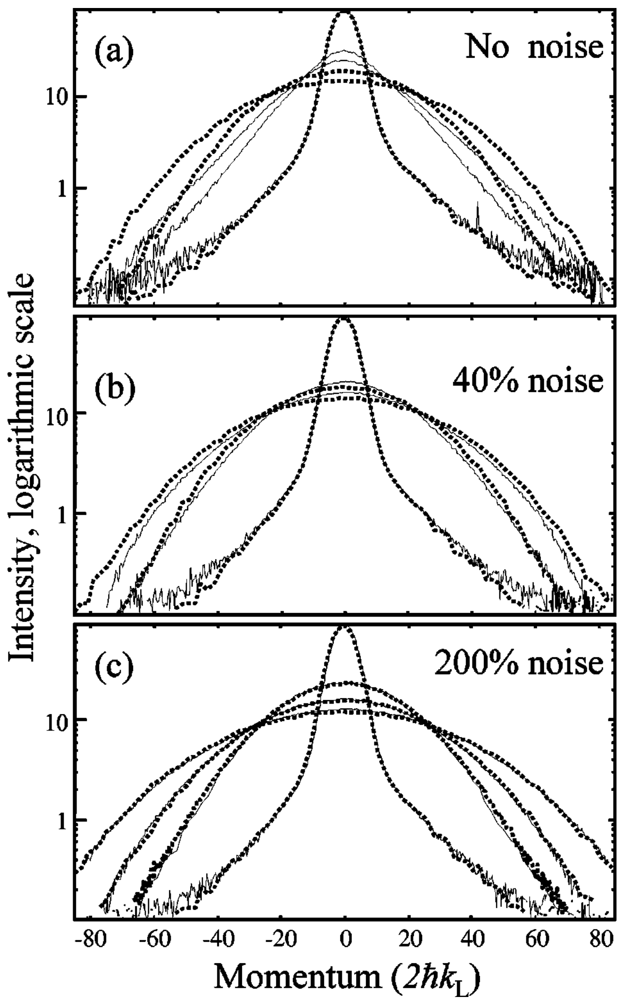

FIG. 2. Experimentally observed (thin solid) and classically simulated (dotted) momentum distributions for three values of amplitude noise. In all three plots, the narrowest distribution represents the initial condition, while each consecutively broader line corresponds to 40 and 80 kicks (a), (b) or (c) 20, 40, and 80 kicks (c). Note the logarithmic intensity scale. $K = 11.2$.

represented by the measured (solid lines) and calculated (dotted lines) momentum distributions after 0, 20 [Fig. 2(c) only], 40, and 80 kicks. The initial conditions for the numerical analysis have been chosen to accurately describe our experimental initial distribution, which consists of a central Gaussian part and a broad pedestal. In the absence of noise [Fig. 2(a)], the line shapes are clearly exponential, in contrast to the classical predictions. For moderate amplitude noise, the observed distributions become Gaussian, though not quite as broad as the calculated ones [Fig. 2(b)]. Only in the strong noise regime (~50% and higher), do the observed distributions coincide with the calculated profiles, attesting to the classical behavior. At our maximum noise level of 200%, the agreement with classical theory is equally good at each step of the time evolution, suggesting a short decoherence time [Fig. 2(c)].

The transformation of the quantum exponential distributions into the classical Gaussians is shown in Fig. 3. Here, the observation and the calculation after 80 kicks are shown, while scanning the noise level from zero to maximum. As can be seen in Fig. 3(a), even weak noise destroys the localization drastically, pushing the observed momentum distribution toward a Gaussian profile, whereas the classical distributions are less affected. However, only when strong noise is applied do we have an exact correspondence between the curves, as seen from the insets.

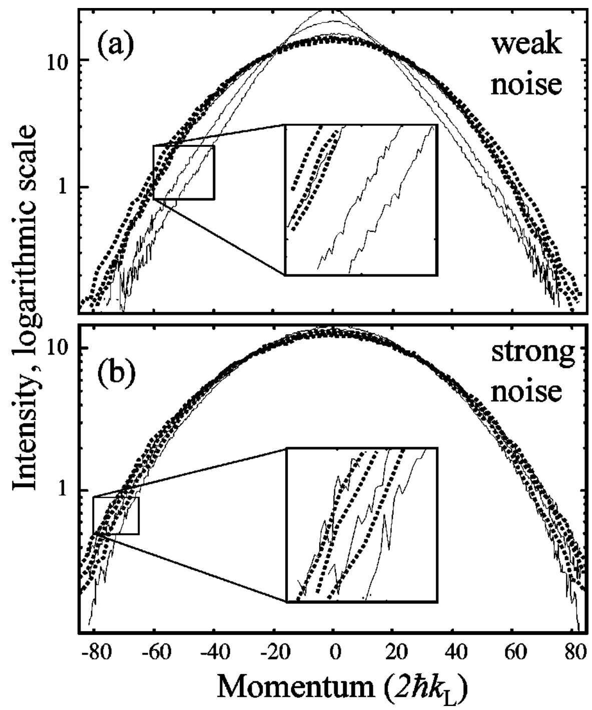

FIG. 3. Experimentally observed (thin solid) and classically simulated (dotted) momentum distributions for the interaction time of 80 kicks. Each consecutively broader line corresponds to higher noise level: (a) 0%, 20%, and 40%; (b) 60%, 100%, and 200%. Note the logarithmic intensity scale (both for main and magnified plots). $K = 11.2$.

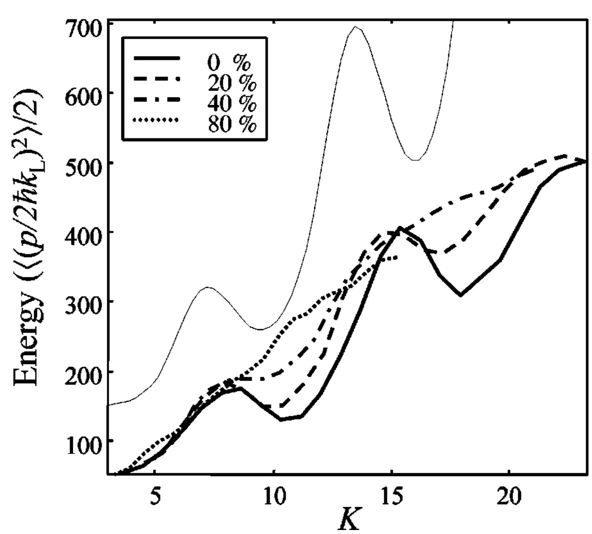

FIG. 4. Experimentally measured energy versus stochasticity parameter for an interaction time of 35 kicks and different levels of amplitude noise (thick lines). Each curve consists of 25 data points, equally spaced in K, and averaged over 10 different realizations of noise. The thin line at the top represents the classical prediction without noise (shifted up for clarity). Note that the experimental curves in this plot show the diminished energy growth at high values of K due to the systematic effects mentioned in the text.

An interesting comparison of the vulnerability of quantum and classical ''memory'' with respect to amplitude noise may be made by monitoring the energy of the system as a function of the stochasticity parameter K. Classical calculations of the diffusion coefficient D_{cl} show an oscillatory dependence on K,

$$D_{\mathrm{cl}}(K)=\frac{K^2}{2}\left[\frac{1}{2}-J_2(K)+J_2^2(K)+O(K^{-3/2})\right], \qquad (2)$$

where $J_2(K)$ is an ordinary Bessel function. The origin of these oscillations is in the short-time kick-to-kick correlations, which influence the system dynamics even for arbitrarily large K [16]. In our earlier work, we observed the signature of such behavior in the quantum kicked rotor [15]. It has been shown that the energy growth in the quantum case oscillates as Eq. (2) with stochasticity parameter K replaced by $K_{\mathrm{q}}=K[\sin(\hbar/2)/(\hbar/2)]$ in the arguments of the Bessel functions [17].

In Fig. 4 the experimentally measured energy after 35 kicks is shown versus the stochasticity parameter K. The oscillatory behavior is clearly seen at the low noise levels (0% and 20%) and is completely washed out when the amplitude noise exceeds 50%. The classical formula (2) is depicted by a thin solid line on top of the measured curves for comparison. These results show that the short-time correlations, modified by quantum effects, ''prevent'' an exact classical behavior even when the long-time quantum correlations, responsible for dynamical localization, are destroyed. Indeed, Gaussian distributions in the 20% and 40% noise cases exhibit diffusive dynamics [see Fig. 3(a)], whereas the shifted (with respect to the classical) oscillations in the energy growth reflect a clear discrepancy between quantum and classical physics. This seemingly surprising result stems from the quantum scaling (K_{q}/K), which persists even in the noise-driven quantum system [18], and makes the destruction of the short-time correlations a necessary condition for the recovery of classical behavior.

In conclusion, the effect of amplitude noise on the quantum kicked rotor has been studied experimentally. We have performed a careful comparison of the observed energies and momentum distributions with the numerical simulations of the classical system for the same amount of noise, taking into account the dominant systematic effects of the experiment. We show that even if weak noise destroys long-time quantum coherences (and thus, dynamical localization), the short-time quantum correlations may persist, resulting in diffusive but not classical behavior. We find that strong amplitude noise is able to drive the quantum system to the classical limit, in the sense that the momentum distributions are experimentally indistinguishable from the classical predictions to within the precision of our measurements.

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